Naloga 1 *V* $\mathbb{R}^5$ *sta dana afina podprostora* P_1 *in* P_2 *s sistemi enačb:*

$$P_1: \begin{aligned} x_1 - 5\,x_2 + 4\,x_3 + 3\,x_4 + 2\,x_5 &= 1 \\ 7\,x_1 - x_2 - x_3 + 2\,x_4 + 3\,x_5 &= -2 \end{aligned} \qquad P_2: \begin{aligned} 2\,x_1 + 2\,x_2 + 2\,x_3 + 3\,x_4 - 3\,x_5 &= 2 \\ 14\,x_1 + 9\,x_3 + 14\,x_4 - 4\,x_5 &= 3 \end{aligned}$$

Izračunaj baze in dimenzije prostorov P_1, P_2, $P_1 + P_2$ *in* $P_1 \cap P_2$.

Rešitev: $\dim(P_1) = 3$, $\dim(P_2) = 3$, $\dim(P_1+P_2) = 5$, $\dim(P_1 \cap P_2) = -1$ $\mathcal{B}(P_1) = \{(-3,-1,5,0,0), (-9,-5,14,0,0), (-2,-1,0,2,0), (4,17,0,0,14)\}$,

$\mathcal{B}(P_2) = \{(0,-2,-1,-1,-1), (9,29,34,0,0), (-7,19,0,34,0), (-13,11,0,0,34)\}$, $\mathcal{B}(P_1 \cap P_2) = \emptyset$,

$\mathcal{B}(P_1 + P_2) = \{(0,0,0,0,0), (1,0,0,0,0), (0,1,0,0,0), (0,0,1,0,0), (0,0,0,1,0), (0,0,0,0,1)\}$.

Naloga 2 *V* $\mathbb{R}^6$ *imamo afina podprostora* $P_1 : 4\,x_1 - 2\,x_2 - 2\,x_3 + x_4 + x_5 + 2\,x_6 = -1$, $x_1 - 2\,x_2 + 2\,x_3 - x_4 + 3\,x_5 + 3\,x_6 = 1$, $2\,x_1 + 5\,x_2 + 3\,x_3 + 2\,x_4 - x_5 - x_6 = 2$ *in* $P_2 : (x_1, x_2, x_3, x_4, x_5, x_6) = (2,4,1,1,0,2) + t\,(2,3,2,1,0,-1)$; $t \in \mathbb{R}$. *Izračunaj baze in dimenzije prostorov* P_1, P_2, $P_1 + P_2$ *in* $P_1 \cap P_2$.

Rešitev: $\dim(P_1) = 3$, $\dim(P_2) = 1$, $\dim(P_1 + P_2) = 5$, $\dim(P_1 \cap P_2) = -1$ $\mathcal{B}(P_2) = \{(2,4,1,1,0,2), (2,3,2,1,0,-1)\}$,
$\mathcal{B}(P_1) = \{(1/21, 5/84, 15/28, 0, 0, 0), (-1/3, -5/12, 1/4, 1, 0, 0), (-5/21, 59/84, -19/28, 0, 1, 0), (-3/7, 5/7, -4/7, 0, 0, 1)\}$, $\mathcal{B}(P_1 + P_2) =$
$\{(2,4,1,1,0,2), (-41/21, -331/84, -13/28, -1, 0, -2), (2,3,2,1,0,-1), (-1/3, -5/12, 1/4, 1, 0, 0), (-5/21, 59/84, -19/28, 0, 1, 0), (-3/7, 5/7, -4/7, 0, 0, 1)\}$,
$\mathcal{B}(P_1 \cap P_2) = \emptyset$.

Naloga 3 *Naj bosta* P_1 *in* P_2 *afina podprostora v* $\mathbb{R}^4$ *podana z*

$$P_1: \begin{aligned} x_1 &= 1 + t_1 \\ x_2 &= 1 + 11\,t_2 \\ x_3 &= 1 - 15\,t_1 \\ x_4 &= -1 + 18\,t_1 - 12\,t_2 \end{aligned} \qquad P_2: \begin{aligned} 2\,x_1 - x_2 + 3\,x_3 - 7\,x_4 &= 5 \\ 6\,x_1 - 3\,x_2 + x_3 - 4\,x_4 &= 7 \\ 4\,x_1 - 2\,x_2 + 14\,x_3 - 31\,x_4 &= 18 \end{aligned}$$

Izberi koordinatni sistem za $P_1 \cap P_2$.

Rešitev: $(3361/3378, -109/3378, 1211/1126, 20/563)$

Naloga 4 *Zapiši enačbo premice, ki vsebuje točko* $B(6,5,1,-1)$ *in seka ravnini* P_1 *in* P_2.

$$P_1: \begin{aligned} -x_1 + 2\,x_2 + x_3 &= 1 \\ x_1 + x_4 &= 1 \end{aligned} \qquad P_2: \begin{aligned} x_1 &= 4 + t \\ x_2 &= 4 + 2\,t \\ x_3 &= 5 + 3\,t \\ x_4 &= 4 + 4\,t \end{aligned}$$

Rešitev: $(6,5,1,-1) + s\,(-3,-3,1,1)$.

Naloga 5 *V* $\mathbb{R}^4$ *sta podana afina podprostora* P_1 *in* P_2. *Izračunaj dimenzijo prostorov* P_1, P_2, $P_1 \cap P_2$ *in* $P_1 + P_2$.

$$P_1: \begin{aligned} 3\,x_1 + 2\,x_2 + 2\,x_3 + 3\,x_4 &= 2 \\ 2\,x_1 + 3\,x_2 + 2\,x_3 + 5\,x_4 &= 3 \end{aligned} \qquad P_2: \begin{aligned} 2\,x_1 + 2\,x_2 + 3\,x_3 + 4\,x_4 &= 5 \\ 5\,x_1 - x_2 + 3\,x_3 - 5\,x_4 &= 2 \end{aligned}$$

Rešitev: $\dim(P_1) = 2$, $\dim(P_2) = 2$, $\dim(P_1 \cap P_2) = 1$, $\dim(P_1 + P_2) = 3$.

$\mathcal{B}(P_1) = \{(-6/7, 1/7, 15/7, 0), (-2/5, -2/5, 1, 0), (4/5, -11/5, 0, 1)\}$, $\mathcal{B}(P_2) = \{(-6/7, 1/7, 15/7, 0), (-3/4, -3/4, 1, 0), (1/2, -5/2, 0, 1)\}$,

$\mathcal{B}(P_1 \cap P_2) = \{(-6/7, 1/7, 15/7, 0), (8/7, -13/7, -6/7, 1)\}$, $\mathcal{B}(P_1 + P_2) = \{(-6/7, 1/7, 15/7, 0), (-2/5, -2/5, 1, 0), (4/5, -11/5, 0, 1), (-3/4, -3/4, 1, 0)\}$.

Naloga 6 *V* $\mathbb{R}^4$ *sta podana afina podprostora* P_1 *in* P_2. *Izračunaj dimenzijo prostorov* P_1, P_2, $P_1 \cap P_2$ *in* $P_1 + P_2$.

$$P_1: \begin{aligned} 2\,x_1 + 3\,x_2 + 4\,x_3 + 5\,x_4 &= 6 \\ 6\,x_1 + 5\,x_2 + 4\,x_3 + 3\,x_4 &= 2 \end{aligned} \qquad P_2: \begin{aligned} x_1 &= 1 - t_1 \\ x_2 &= 1 + 2\,t_1 + t_2 \\ x_3 &= 1 - 2\,t_1 + 2\,t_2 \\ x_4 &= 1 + t_1 + t_2 \end{aligned}$$

Rešitev: $\dim(P_1) = 2$, $\dim(P_2) = 2$, $\dim(P_1 \cap P_2) = 0$, $\dim(P_1 + P_2) = 4$.

$\mathcal{B}(P_1) = \{(-3,4,0,0),(1,-2,1,0),(2,-3,0,1)\}$, $\mathcal{B}(P_2) = \{(1,1,1,1),(-1,2,-2,1),(0,1,2,1)\}$, $\mathcal{B}(P_1 \cap P_2) = \{(-3,33/4,-17/2,17/4)\}$,

$\mathcal{B}(P_1 + P_2) = \{(0,0,0,0),(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}$.

Naloga 7 *V* $\mathbb{R}^4$ *sta podana afina podprostora* P_1 *in* P_2. *Izračunaj dimenzijo prostorov* P_1, P_2, $P_1 \cap P_2$ *in* $P_1 + P_2$.

$$P_1: \begin{aligned} x_1 &= 1 + 2\,t_1 \\ x_2 &= 3 + 2\,t_2 \\ x_3 &= 5 + 4\,t_2 \\ x_4 &= 4 + 3\,t_1 + 2\,t_2 \\ x_5 &= 2 + t_1 + 2\,t_2 \end{aligned} \qquad P_2: \begin{aligned} x_1 &= -6 + t \\ x_2 &= 2 + 3\,t \\ x_3 &= 2 + 7\,t \\ x_4 &= -2 + 5\,t \\ x_5 &= -3 + 3\,t \end{aligned}$$

Rešitev: $\dim(P_1) = 2$, $\dim(P_2) = 2$, $\dim(P_1 \cap P_2) = -1$, $\dim(P_1 + P_2) = 4$.

$\mathcal{B}(P_1) = \{(1,3,5,4,2),(2,0,0,3,1),(0,2,4,2,2)\}$, $\mathcal{B}(P_2) = \{(-6,2,2,-2,-3),(0,3,7,5,3)\}$, $\mathcal{B}(P_1 \cap P_2) = \emptyset$, $\mathcal{B}(P_1 + P_2) = \{(1,3,5,4,2),(0,0,0,15,-1),(5,0,0$

Naloga 8 *Poišči afino preslikavo* $A : \mathbb{R}^2 \to \mathbb{R}^2$, *ki po vrsti preslika točke* $(1,2)$, $(0,4)$ *in* $(-1,1)$ *v točke* $(11,-3)$, $(20,4)$ *in* $(4,6)$. *Poišči negibne točke preslikave* A.

Rešitev:

$$A = \begin{bmatrix} 1 & 5 & 0 \\ -5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \mathcal{N}(A) = \emptyset$$

Naloga 9 *Poišči afino preslikavo* $A : \mathbb{R}^3 \to \mathbb{R}^3$, *ki po vrsti preslika točke* $(2,-1,0)$, $(0,1,2)$, $(3,1,1)$ *in* $(-1,0,1)$ *v točke* $(1,1,0)$, $(0,-1,1)$, $(2,2,1)$ *in* $(-1,3,-1)$. *Poišči negibne točke preslikave* A.

Rešitev:

$$A = \begin{bmatrix} 3/4 & 0 & 1/4 & -1/2 \\ -3/2 & 5 & -15/2 & 9 \\ 3/4 & -1 & 9/4 & -5/2 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathcal{N}(A) = \{(-6,-12,-4)\}$$

Naloga 10 *Naj bo* $A : \mathbb{R}^4 \to \mathbb{R}^4$ *afina preslikava, ki preslika točko* $(1,2,-1,0)$ *v* $(3,0,-1,2)$. *Njen linearen del naj bo* $\begin{bmatrix} 2 & 3 & -1 & 2 \\ -1 & 0 & 1 & 2 \\ 0 & 1 & -1 & 4 \\ 4 & 1 & 2 & 1 \end{bmatrix}$. *Kam preslika* A *točki* $B(-1,0,4,2)$ *in* $C(3,3,1,-1)$? *Poišči njene negibne točke.*

Rešitev:

$$A = \begin{bmatrix} 2 & 3 & -1 & 2 & -6 \\ -1 & 0 & 1 & 2 & 2 \\ 0 & 1 & -1 & 4 & -4 \\ 4 & 1 & 2 & 1 & -2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad A(B) = (-8,11,0,4), \quad A(C) = (6,-2,-6,14), \quad \mathcal{N}(A) = \{(1/2,7/5,-7/10,3/10)\}.$$

Naloga 11 *Ali obstaja afina preslikava* $A : \mathbb{R}^4 \to \mathbb{R}^4$, *ki po vrsti preslika točke* $a(1,1,1,1)$, $b(2,3,2,3)$, $c(3,2,3,2)$, $d(1,2,2,2)$ *in* $e(1,0,2,0)$ *v točke* $a'(-1,1,-1,1)$, $b'(0,4,0,4)$, $c'(2,2,2,2)$, $d'(-1,2,0,3)$ *in* $e'(0,-3,1,-2)$*?*

Rešitev: Točke a, b, c, d in e so afino odvisne, točke a, b, c in d pa so afino neodvisne. Izkaže se, da je $e = \frac{2}{3}a - \frac{4}{3}b + \frac{2}{3}c + d$. Torej mora biti $A(e) = A(\frac{2}{3}a - \frac{4}{3}b + \frac{2}{3}c + d) = \frac{2}{3}a' - \frac{4}{3}b' + \frac{2}{3}c' + d' = (-\frac{1}{3}, -\frac{4}{3}, \frac{2}{3}, -\frac{1}{3}) \neq e'$. Zato taka afina preslikava ne obstaja.

Naloga 12 *Naj bo* $A : \mathbb{R}^3 \to \mathbb{R}^3$ *afina preslikava. Njen linearen del je* $\begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 1 \\ -4 & 1 & 2 \end{bmatrix}$ *in* $A(1,-1,2) = (0,3,1)$*. Kam* A *preslika premico* $(1,2,-1) + t(-3,2,1)$*?*

Rešitev: $(-12, 3, -2) + s(-5, 3, 16)$.

Naloga 13 *Poišči kako afino preslikavo* $A : \mathbb{R}^4 \to \mathbb{R}^4$, *ki po vrsti preslika* $(2,-1,3,-2)$, $(3,1,6,-1)$ *in* $(5,1,4,1)$ *v* $(1,-2,3,5)$, $(2,1,8,7)$ *in* $(3,2,10,-6)$ *in premico* $(2,0,4,-1) + u(0,1,1,0)$ *v premico* $(1,-1,5,-2) + v(0,2,3,-3)$*.*

Rešitev:

$$A = \begin{bmatrix} 3/4 & -1/4 & 1/4 & 0 & -3/2 \\ 3/4 & 11/4 & -3/4 & -1 & -1/2 \\ 1 & 4 & -1 & -1 & 6 \\ 19/4 & -41/4 & 29/4 & -4 & -89/2 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B(s,t) = \begin{bmatrix} 3/4 & -1/4 & 1/4 & 0 & -3/2 \\ s*(-2)+3/4 & t*4+(-5/4) & t*(-2)+5/4 & s*2+t*(-2)+1 & s*8+t*6+(-13/2) \\ s*(-3)+1 & t*6+(-2) & t*(-3)+2 & s*3+t*(-3)+2 & s*12+t*9+(-3) \\ s*3+19/4 & t*(-6)+(-17/4) & t*3+17/4 & s*(-3)+t*3+(-7) & s*(-12)+t*(-9)+(-71/2) \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\{B(s,t) | s,t \in \mathbb{R},\quad t \neq 0\}$ so vse rešitve in $A = B(0,1)$.

Naloga 14 *Poišči afino preslikavo* $A : \mathbb{R}^3 \to \mathbb{R}^3$, *ki po vrsti preslika točke* $(2,5,1)$, $(3,5,1)$, $(2,6,1)$ *in* $(2,5,2)$ *v točke* $(3,7,3)$, $(6,11,6)$, $(5,17,9)$ *in* $(0,-5,-4)$*. Poišči negibne točke preslikave* A*.*

Rešitev:

$$A = \begin{bmatrix} 3 & 2 & -3 & -10 \\ 4 & 10 & -12 & -39 \\ 3 & 6 & -7 & -26 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \mathcal{N}(A) = \{(0,-1,-4)\}$$

Naloga 15 *Poišči negibne točke afine preslikave* $A : \mathbb{R}^3 \to \mathbb{R}^3$, *podane z* $\begin{bmatrix} -1 & -3 & 4 & -1 \\ 4 & -9 & 8 & -14 \\ 6 & -7 & 5 & -13 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Rešitev: $\mathcal{N}(A) = \{(1,-1,0) + t(1,2,2) | t \in \mathbb{R}\}$.

Naloga 16 *Poišči vse afine preslikave* $\mathbb{R}^4 \to \mathbb{R}^4$, *ki po vrsti preslikajo* $(4,4,4,-\frac{5}{2})$, $(5,-4,0,3)$, $(4,4,6,8)$ *v* $(1,3,2,-2)$, $(4,-2,0,0)$, $(-3,10,6,2)$, *premico* $(9,-2,4,7) + s(4,4,5,2)$ *pa v premico* $(3,0,2,3) + t(2,-6,-3,2)$ *in pri tem ohranja njuno presečišče.*

Rešitev: Presečišče premic je $(5,-6,-1,5)$, družina iskanih preslikav pa

$$\left\{ \begin{bmatrix} 119 & 349/2 & -254 & 48 & -37 \\ -614 & -906 & 1316 & -250 & 194 \\ -158 & -465/2 & 338 & -64 & 52 \\ 742 & 2197/2 & -1594 & 304 & -228 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} + t \begin{bmatrix} 20 & 29 & -42 & 8 & -8 \\ -60 & -87 & 126 & -24 & 24 \\ -30 & -87/2 & 63 & -12 & 12 \\ 20 & 29 & -42 & 8 & -8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad t \in \mathbb{R}\backslash\{0\} \right\}$$