

### Radio $k$ -labelings of $D(1,2,\dots,t)$

Regular font: previously known results

Bold font: Theorem 3

Orange color: lower bound obtained by ILP

Red color: lower bound obtained by SAT solver

$t \setminus k$	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	6	12	20	30	42	56	65-72	82-90	101-110	122-132	145-156	170-182	197-210	226-240
3	8	17	28	43	59-60	74-81	97-104	122- <b>131</b>	151-160	182- <b>193</b>	217-228	254- <b>267</b>	295-308	338- <b>353</b>
4	10	22	36	55-56	76-78	99- <b>106</b>	129-136	163- <b>172</b>	201-210	243- <b>254</b>	289- <b>300</b>	339- <b>352</b>	393-406	451- <b>466</b>
5	12	27	44	67-69	94-96	123-131	161-168	203- <b>213</b>	251-260	303- <b>315</b>	361-372	423- <b>437</b>	491-504	563- <b>579</b>
6	14	32	52	80-82	109-114	148- <b>156</b>	193-200	244- <b>254</b>	301-310	364- <b>376</b>	433-444	508- <b>522</b>	589-602	676- <b>692</b>
7	16	37	60	92-95	127-132	172- <b>181</b>	225-232	284- <b>295</b>	351-360	424- <b>437</b>	505-516	592- <b>607</b>	687-700	788- <b>805</b>
8	18	42	68	105-108	145-150	197- <b>206</b>	257-264	325- <b>336</b>	401-410	485- <b>498</b>	577-588	677- <b>692</b>	785-798	901- <b>918</b>
9	20	47	75-76	116-121	163-168	221- <b>231</b>	289-296	365- <b>377</b>	451-460	545- <b>559</b>	649-660	761- <b>777</b>	883-896	1013- <b>1031</b>
10	22	46- <b>52</b>	81-84	126- <b>134</b>	181-186	246- <b>256</b>	321-328	406- <b>418</b>	501-510	606- <b>620</b>	721-732	846- <b>862</b>	981-994	1126- <b>1144</b>
11	24	50- <b>57</b>	89-92	138- <b>147</b>	199-204	270- <b>281</b>	353-360	446- <b>459</b>	551-560	666- <b>681</b>	793-804	930- <b>947</b>	1079-1092	1238- <b>1257</b>
12	26	55- <b>62</b>	97-100	151- <b>160</b>	217-222	295- <b>306</b>	385-392	487- <b>500</b>	601-610	727- <b>742</b>	865-876	1015- <b>1032</b>	1177-1190	1351- <b>1370</b>
13	27-28	59- <b>67</b>	105-108	163- <b>173</b>	235-240	319- <b>331</b>	417-424	527- <b>541</b>	651-660	787- <b>803</b>	937-948	1099- <b>1117</b>	1275-1288	1463- <b>1483</b>
14	29-30	64- <b>72</b>	113-116	176- <b>186</b>	253-258	344- <b>356</b>	449-456	568- <b>582</b>	701-710	848- <b>864</b>	1009-1020	1184- <b>1202</b>	1373-1386	1576- <b>1596</b>
15	31-32	68- <b>77</b>	121-124	188- <b>199</b>	271-276	368- <b>381</b>	481-488	608- <b>623</b>	751-760	908- <b>925</b>	1081-1092	1268- <b>1287</b>	1471-1484	1688- <b>1709</b>

**Lower bounds  $D(1,2,\dots,t)$  (SAT solver):**

k=2, t=10: 13 nodes graph was UNSAT using labels 0-21 (947 seconds, 10 threads)

k=2, t=11: 14 nodes graph was UNSAT using labels 0-23 (12168 seconds, 10 threads)

k=2, t=12: 15 nodes graph was UNSAT using labels 0-25 (196890 seconds, 32 threads)

### Radio $k$ -labelings of $D(t-1,t)$

Lower and upper bound or exact value

Regular font – previously known results

Bold font – Upper bound obtained by a construction

Green color – Lower bound obtained by Theorem 7

Orange color – Lower bound obtained by ILP

Red color – Lower or upper bound obtained by SAT solver (constructions given below)

Purple color – Lower bound obtained by SAT solver, upper bound obtained by a construction

$t \backslash k$	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	6	12	20	30	42	56	66-72	83-90	92-110	111-132	134-156	157-182	184-210	211-240
3	6	14	28	40	55-60	71-78	85-104	117-128	131-160	161-190	193-228	229-264	267-308	309-350
4	7	14	27	42-48	60-70	82-99	106-131	130-166	154-204	191-250	232-295	277-348	326-406	379-462
5	6	13	26	42-50	61-78	88-111	117-150	141-191	177-244	221-302	271-358	325-415	385-492	449-568
6	6	14	27-29	36-51	54-69	80-108	113-153	148-208	184-264	235-332	290-400	353-480	420-560	495-652
7	7	13	27	38-48	55-74	82-109	107-148	145-217	193-283	249-354	311-438	381-520	457-626	541-720
8	7	13	26	35-48	58-73	76-120	103-144	145-211	193-280	249-357	316-447	389-554	474-660	563-783
9	6	12	27	37-53	53-70	81-109	96-160	137-217	190-260	249-354	321-448	401-572	491-694	589-820
10	6	13	27	30-50	40-72	53-120	89-154	133-216	187-258	249-353	321-435	401-576	491-696	595-832
11	7	12	26	30-48	40-74	43-110	74-148	121-226	172-270	239-358	310-426	397-558	491-692	601-840
12	6	13	27	30-50	40-72	45-107	65-148	109-208	163-284	229-381	305-432	393-546	491-651	601-824
13	6	12	27	30-54	40-70	45-110	52-159	89-210	146-265	207-370	288-442	373-564	478-660	587-819
14	7	12	27	30-48	40-72	45-113	39-156	77-222	129-275	193-354	271-432	361-564	465-671	581-822
15	7	12	27	30-47	40-74	45-120	22-152	61-216	104-277	171-359	242-448	337-558	436-666	559-843

**D(2,3)**

k	Upper b.	Construction (pattern)
10	160	$b(i)=50i \bmod 161$
11	190	$b(i)=61i \bmod 193$
12	228	$b(i)=72i \bmod 229$
13	264	$b(i)=85i \bmod 268$
14	308	$b(i)=98i \bmod 309$
15	350	$b(i)=113i \bmod 354$

**D(3,4)**

k	Upper b.	Construction (pattern)
10	204	$b(i)=100i \bmod 208$
11	250	$b(i)=122i \bmod 253$
12	295	$b(i)=145i \bmod 300$
13	348	$b(i)=348i \bmod 349$
14	406	$b(i)=98i \bmod 407$
15	462	$b(i)=183i \bmod 464$

**D(4,5)**

k	Upper b.	Construction (pattern)
7	111	$b(i)=54*i \bmod 112$
10	244	$b(i)=71i \bmod 245$
11	302	$b(i)=137i \bmod 303$
12	358	$b(i)=50i \bmod 359$
13	415	$b(i)=205i \bmod 420$
14	492	$b(i)=72i \bmod 493$
15	568	$b(i)=208i \bmod 569$

**D(5,6)**

k	Upper b.	Construction (pattern)
4	29	0 3 16 19 2 5 19 22 5 8 21 24 8 11 24 27 10 13 26 29 13 16 29 1 15 18 1 4 18 21 4 7 20 23 6 9 23 26 9 12 25 28 11 14 28 0 14 17
5	51	$b(i) = 25*i \text{ mod } 52$
10	264	$b(i) = 87i \text{ mod } 267$
11	332	$b(i) = 124i \text{ mod } 333$
12	400	$b(i) = 132i \text{ mod } 404$
13	480	$b(i) = 59i \text{ mod } 481$
14	560	$b(i) = 185i \text{ mod } 565$
15	652	$b(i) = 83i \text{ mod } 653$

**D(6,7)**

k	Upper b.	Construction (pattern)
4	27	0 15 1 16 3 18 5 20 6 21 8 23 10 25 12 27 13 0 15 2 17 3 18 5 20 7 22 8 23 10 25 12 27 14 0 15 2 17 4 19 5 20 7 22 9 24 11 25 12 27 14 1 16 2 17 4 19 6 21 7 22 9 24 11 26 13 27 14 1 16 3 18 4 19 6 21 8 23 9 24 11 26 13
9	217	$b(i) = 65*i \text{ mod } 218$
10	283	$b(i) = 93i \text{ mod } 284$
11	354	$b(i) = 117i \text{ mod } 357$
12	438	$b(i) = 131i \text{ mod } 439$
13	520	$b(i) = 172i \text{ mod } 524$
14	626	$b(i) = 212i \text{ mod } 627$
15	720	$b(i) = 245i \text{ mod } 728$

**D(7,8)**

k	Upper b.	Construction (pattern)
8	144	$b(i) = 23*i \text{ mod } 145$
10	280	$b(i) = 77i \text{ mod } 281$

11	357	$b(i)=357i \text{ mod } 358$
12	447	$b(i)=111i \text{ mod } 450$
13	554	$b(i)=137i \text{ mod } 555$
14	660	$b(i)=164i \text{ mod } 664$
15	783	$b(i)=213i \text{ mod } 784$

**D(8,9)**

k	Upper b.	Construction (pattern)
8	160	$b(i)=59*i \text{ mod } 161$
9	217	$b(i)=80*i \text{ mod } 218$
10	260	$b(i)=25i \text{ mod } 261$
11	354	$b(i)=34i \text{ mod } 355$
12	448	$b(i)=104i \text{ mod } 449$
13	572	$b(i)=132i \text{ mod } 574$
14	694	$b(i)=172i \text{ mod } 695$
15	820	$b(i)=204i \text{ mod } 824$

**D(9,10)**

k	Upper b.	Construction (pattern)
2	6	$b(i)=i \text{ mod } 7$
3	13	0 5 9 13 5 2 13 11 2 6 10 1 6 10 8 6 3 7 12 3 7 11 3 0 11 9 0 4 8 13 4 8 12 4 1 12 10 1 5 9
4	27	0 13 2 15 25 10 23 12 25 7 20 9 22 4 17 6 19 1 14 3 16 26 11 0 13 26 8 21 10 23 5 18 7 20 2 15 4 17 27 12 1 14 27 9 22 11 24 6 19 8 21 3 16 5 18
5	50	$b(i)=22i \text{ mod } 51$
6	72	$b(i)=28i \text{ mod } 73$
7	120	$b(i)=34i \text{ mod } 121$
8	154	$b(i)=24i \text{ mod } 155$
9	216	$b(i)=32i \text{ mod } 218$
10	258	$b(i)=126i \text{ mod } 260$
11	353	$b(i)=43i \text{ mod } 354$

12	435	$b(i)=91i \pmod{436}$
13	576	$b(i)=120i \pmod{577}$
14	696	$b(i)=150i \pmod{698}$
15	832	$b(i)=60i \pmod{833}$

**D(10,11)**

k	Upper b.	Construction (pattern)
2	7	$b(i)=i \pmod{8}$
3	12	0 2 3 5 7 9 11 0 2 4 6 8 10 12 1 3 4 6 8 10 12 1 3 5 7 9 11 0 2 4 6 8 10 12 1 4 6 8 10 12 1 3 5 7 9 11 0 2 5 7 9 11 0 2 4 6 8 10 12 1 3 5 7 9 11
4	26	$b(i)=13i \pmod{27}$
5	48	$b(i)=12i \pmod{49}$
6	74	$b(i)=6i \pmod{76}$
7	110	$b(i)=38i \pmod{111}$
8	148	$b(i)=51i \pmod{149}$
9	226	$b(i)=26i \pmod{227}$
10	270	$b(i)=67i \pmod{271}$
11	358	$b(i)=46i \pmod{360}$
12	426	$b(i)=23i \pmod{427}$
13	558	$b(i)=15i \pmod{559}$
14	692	$b(i)=293i \pmod{693}$
15	840	$b(i)=356i \pmod{842}$

**D(11,12)**

k	Upper b.	Construction (pattern)
2	6	$b(i)=i \pmod{7}$
3	13	0 10 8 6 4 1 4 6 8 10 12 9 7 5 3 0 9 11 13 0 2 0 6 4 2 12 10 12 6 8 10 12 5 3 1 11 9 7 5 3 0 5 7 9 11 13 8 6 4 2 8 10 12 1 3 1 7 5 3 1 13 11 13 7 9 11 13 4 2
4	27	$b(i)=3i \pmod{28}$
5	50	$b(i)=10i \pmod{52}$

6	72	$b(i)=21i \pmod{73}$
7	107	$b(i)=17i \pmod{108}$
8	148	$b(i)=11i \pmod{149}$
9	208	$b(i)=71i \pmod{209}$
10	284	$b(i)=14i \pmod{286}$
11	381	$b(i)=169i \pmod{382}$
12	432	$b(i)=192i \pmod{434}$
13	546	$b(i)=242i \pmod{547}$
14	651	$b(i)=49i \pmod{652}$
15	824	$b(i)=62i \pmod{825}$

**D(12,13)**

k	Upper b.	Construction (pattern)
2	6	$b(i)=2i \pmod{7}$
3	12	0 2 4 6 8 10 12 10 12 10 12 1 4 6 8 10 12 1 3 5 7 1 3 5 7 9 11 0 2 4 6 0 2 4 6 8 10 12 1 3 5 7 9 11 9 11 9 11 0 2 5 7 9 11 0 2 4 6 0 2 4 6 8 10 12 1 3 5 7 1 3 5 7 9 11
4	27	0 14 27 12 26 11 24 10 23 8 22 7 20 5 19 4 18 3 16 1 15 0 14 27 12 25 11 24 9 23 8 21 7 20 5 19 4 17 2 16 1 15 0 13 27 12 25 10 24 9 22 8 21 6 20 5 18 3 17 2 16 1 14 27 13 26 11 25 10 23 9 22 7 21 6 19 5 18 3 17 2 15
5	54	$b(i)=22i \pmod{56}$
6	70	$b(i)=14i \pmod{71}$
7	110	$b(i)=38i \pmod{112}$
8	159	$b(i)=11i \pmod{160}$
9	210	$b(i)=23i \pmod{211}$
10	265	$b(i)=29i \pmod{266}$
11	370	$b(i)=22i \pmod{371}$
12	442	$b(i)=101i \pmod{443}$
13	564	$b(i)=262i \pmod{565}$
14	660	$b(i)=177i \pmod{661}$
15	819	$b(i)=249i \pmod{820}$

**D(13,14)**

k	Upper b.	Construction (pattern)
2	7	$b(i)=i \bmod 8$
3	12	0 9 5 0 9 5 1 9 5 1 10 5 1 10 6 1 10 6 2 10 6 2 11 6 2 11 7 2 11 7 3 11 7 3 12 7 3 12 8 3 12 8 4 12 8 4 0 8 4 0 9 4
4	27	0 14 10 6 2 27 21 17 13 27 23 19 15 11 7 3 26 22 18 14 10 6 2 7 3 26 22 27 23 19 15 11 7 3 26 22 18 14 10 6 2 16 12 8 4 0 23 19 15 11 25 21 17 13 9 5 1 24 20 16 12 8 4 0 5 1 24 20 25 21 17 13 9 5 1 24 20 16 12 8 4
5	48	$b(i)=18i \bmod 49$
6	72	$b(i)=12i \bmod 74$
7	113	$b(i)=47i \bmod 114$
8	156	$b(i)=76i \bmod 157$
9	222	$b(i)=31i \bmod 223$
10	275	$b(i)=113i \bmod 276$
11	354	$b(i)=107i \bmod 355$
12	432	$b(i)=127i \bmod 433$
13	564	$b(i)=166i \bmod 566$
14	671	$b(i)=227i \bmod 672$
15	822	$b(i)=235i \bmod 823$

**D(14,15)**

k	Upper b.	Construction (pattern)
2	7	$b(i)=3i \bmod 8$
3	12	0 24 6 8 10 12 1 3 5 8 10 12 1 3 5 7 9 11 0 2 4 7 9 11 0 2 4 6 8 10 12 1 3 6 8 10 12 1 3 5 7 9 11 1 3 5 7 9 11 0 2 4 6 9 11 0 2 4 6 8 10 12 1 3 6 8 10 12 1 3 5 7 9 11 0 2 4 7 9 11 0 2 4 6 8 10
4	27	0 25 1 26 2 27 0 4 1 5 2 6 3 7 4 8 5 9 6 10 7 11 8 12 9 13 10 14 11 15 12 16 13 17 14 18 15 19 16 20 17 21 18 22 19 23 20 24 21 25 22 26 23 27 24 0 25 1 26 2 27 0 4 1 5 2 6 3 7 4 8 5 9 6 10 7 11 8 12 9 13 10 14 11 15 12 16 13 17 14 18 15 19 16 20 17 21 18 22 19 23 20 24 21 25 22 26 23 27 24
5	47	$b(i)=23i \bmod 48$
6	74	$b(i)=26i \bmod 76$
7	120	$b(i)=47i \bmod 121$
8	152	$b(i)=50i \bmod 154$
9	216	$b(i)=89i \bmod 217$

10	277	$b(i)=77i \bmod 278$
11	359	$b(i)=53i \bmod 360$
12	448	$b(i)=108i \bmod 449$
13	558	$b(i)=138i \bmod 559$
14	666	$b(i)=186i \bmod 668$
15	843	$b(i)=235i \bmod 844$

### Lower bounds $D(t-1,t)$ (SAT solver):

k=2, t=10: 23 nodes graph was UNSAT using labels 0-5 (<1 second, 10 threads)  
k=2, t=11: 55 nodes graph was UNSAT using labels 0-6 (<1 second, 10 threads)  
k=2, t=12: 27 nodes graph was UNSAT using labels 0-5 (<1 second, 10 threads)  
k=2, t=13: 29 nodes graph was UNSAT using labels 0-5 (<1 second, 10 threads)  
k=2, t=14: 76 nodes graph was UNSAT using labels 0-6 (<1 second, 10 threads)  
k=2, t=15: 74 nodes graph was UNSAT using labels 0-6 (<1 second, 10 threads)

k=3, t=10: 55 nodes graph was UNSAT using labels 0-12 (24 second, 10 threads)  
k=3, t=11: 36 nodes graph was UNSAT using labels 0-11 (<2 second, 10 threads)  
k=3, t=12: 76 nodes graph was UNSAT using labels 0-12 (37 second, 10 threads)  
k=3, t=13: 45 nodes graph was UNSAT using labels 0-11 (<2 second, 10 threads)  
k=3, t=14: 50 nodes graph was UNSAT using labels 0-11 (<2 second, 10 threads)  
k=3, t=15: 54 nodes graph was UNSAT using labels 0-11 (<2 second, 10 threads)

k=4, t=6: 32 nodes graph was UNSAT using labels 0-26 (3408 seconds, 10 threads)  
k=4, t=7: 52 nodes graph was UNSAT using labels 0-26 (2891 seconds, 10 threads)  
k=4, t=8: 39 nodes graph was UNSAT using labels 0-25 (674 seconds, 10 threads)  
k=4, t=9: 108 nodes graph was UNSAT using labels 0-26 (15380 seconds, 10 threads)  
k=4, t=10: 60 nodes graph was UNSAT using labels 0-26 (9983 seconds, 10 threads)  
k=4, t=11: 53 nodes graph was UNSAT using labels 0-25 (28399 seconds, 10 threads)  
k=4, t=12: 68 nodes graph was UNSAT using labels 0-26 (293658 seconds, 30 threads)  
k=4, t=13: 142 nodes graph was UNSAT using labels 0-26 (18404 seconds, 10 threads)  
k=4, t=14: 80 nodes graph UNSAT using labels 0-26 (81053 seconds, 10 threads)  
k=4, t=15: 190 nodes graph was UNSAT using labels 0-26 (31424 seconds, 10 threads)

k=5, t=10-15: 50 nodes graph was UNSAT using labels 0-29 (less than 10 minutes, 10 threads)  
k=6, t=10-15: 50 nodes graph was UNSAT using labels 0-39 (less than 10 minutes, 10 threads)  
k=7, t=12-15: 50 nodes graph was UNSAT using labels 0-44 (less than 10 minutes, 10 threads)

### Radio $k$ -labelings of $D(1,t)$

Lower and upper bound or exact value

Regular font – previously known results

Bold font – upper bound obtained by a construction

Green color – lower bound obtained by Theorem 6

Red color – lower or upper bound obtained by SAT solver

Purple color – lower bound obtained by SAT solver, upper bound by a construction

$t \setminus k$	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	6	12	20	30	42	56	65-72	82-90	51-110	61-132	73-156	85-182	99-210	113-240
3	6	11	24	33	51-52	61-67	81-96	105-113	92-144	161-171	193-214	229-241	267-284	309-323
4	6	15	26	43	54-63	69-88	95-116	124-150	131-185	195-226	237-270	282-319	332-372	385-429
5	6	13	26	41	49-66	73-91	103-128	137-153	158-204	221-247	271-310	325-345	385-418	449-477
6	7	14	28	41-48	49-72	73-102	105-138	144-182	177-232	239-286	295-344	358-412	426-484	501-562
7	7	12	26	37	44-70	71-99	104-146	145-181	188-246	249-299	312-372	381-421	458-520	541-583
8	7	13	26	36-48	46-75	69-114	103-154	145-205	193-258	252-324	320-396	395-479	479-567	570-664
9	6	11	27-28	32-41	41-74	59-91	96-144	137-189	190-264	250-315	321-418	402-481	491-596	590-681
10	6	13	26	30-52	40-70	53-110	89-150	133-208	187-276	249-354	321-429	401-532	495-632	600-744
11	6	12	26	30-37	40-74	43-97	74-148	121-177	172-262	239-329	310-436	397-513	492-644	602-747
12	6	13	27-28	30-52	40-72	45-107	65-146	109-200	163-272	229-360	305-463	393-564	491-660	601-804
13	7	12	27	30-41	40-78	45-99	52-160	89-193	146-278	207-311	288-426	373-517	478-660	587-775
14	7	12	27	30-48	40-81	45-108	39-148	77-212	129-264	193-344	271-438	361-554	465-684	581-820
15	7	11	27	30-48	40-78	45-103	22-150	61-185	104-282	171-331	242-438	337-493	436-654	559-775

**D(1,3)**

k	Upper b.	Construction (pattern)
7	67	$b(i) = 25*i \text{ mod } 68$
8	96	$b(i) = 18*i \text{ mod } 97$
9	113	$b(i) = 41*i \text{ mod } 114$
10	144	$b(i) = 26i \text{ mod } 146$
11	171	$b(i) = 61i \text{ mod } 172$
12	214	$b(i) = 97i \text{ mod } 215$
13	241	$b(i) = 85i \text{ mod } 242$
14	284	$b(i) = 50i \text{ mod } 286$
15	323	$b(i) = 113i \text{ mod } 324$

**D1,4)**

k	Upper b.	Construction (pattern)
6	63	$b(i) = 23*i \text{ mod } 64$
7	88	$b(i) = 32*i \text{ mod } 89$
9	150	$b(i) = 32*i \text{ mod } 151$
10	185	$b(i) = 59i \text{ mod } 186$
11	226	$b(i) = 72i \text{ mod } 227$
12	270	$b(i) = 94i \text{ mod } 271$
13	319	$b(i) = 111i \text{ mod } 320$
14	372	$b(i) = 55i \text{ mod } 373$
15	429	$b(i) = 83i \text{ mod } 430$

**D(1, 5)**

k	Upper b.	Construction (pattern)
8	128	$b(i) = 50*i \text{ mod } 129$
9	153	$b(i) = 47*i \text{ mod } 154$
10	204	$b(i) = 96i \text{ mod } 207$

11	247	$b(i)=115i \bmod 248$
12	310	$b(i)=122i \bmod 311$
13	345	$b(i)=105i \bmod 346$
14	418	$b(i)=55i \bmod 419$
15	477	$b(i)=63i \bmod 480$

### D(1,6)

k	Upper b.	Construction (pattern)
8	138	$b(i)=26*i \bmod 140$
9	182	$b(i)=34*i \bmod 183$
10	232	$b(i)=89i \bmod 233$
11	286	$b(i)=110i \bmod 288$
12	344	$b(i)=62i \bmod 346$
13	412	$b(i)=74i \bmod 413$
14	484	$b(i)=43i \bmod 485$
15	562	$b(i)=50i \bmod 564$

### D(1,7)

k	Upper b.	Construction (pattern)
6	70	$b(i)=26*i \bmod 71$
7	99	$b(i)=27*i \bmod 100$
9	181	$b(i)=17*i \bmod 182$
10	246	$b(i)=39i \bmod 249$
11	299	$b(i)=47i \bmod 300$
12	372	$b(i)=134i \bmod 373$
13	421	$b(i)=101i \bmod 422$
14	520	$b(i)=168i \bmod 524$
15	583	$b(i)=55i \bmod 584$

### D(1,8)

k	Upper b.	Construction (pattern)
7	114	$b(i)=22*i \text{ mod } 115$
8	154	$b(i)=32*i \text{ mod } 155$
9	205	$b(i)=63*i \text{ mod } 206$
10	258	$b(i)=120*i \text{ mod } 261$
11	324	$b(i)=156*i \text{ mod } 327$
12	396	$b(i)=69*i \text{ mod } 399$
13	479	$b(i)=83*i \text{ mod } 480$
14	567	$b(i)=123*i \text{ mod } 568$
15	664	$b(i)=144*i \text{ mod } 665$

### D(1,9)

k	Upper b.	Construction (pattern)
7	91	$b(i)=33*i \text{ mod } 92$
8	144	$b(i)=19*i \text{ mod } 145$
9	189	$b(i)=89*i \text{ mod } 190$
10	264	$b(i)=75*i \text{ mod } 267$
11	315	$b(i)=93*i \text{ mod } 318$
12	418	$b(i)=51*i \text{ mod } 419$
13	481	$b(i)=29*i \text{ mod } 482$
14	596	$b(i)=244*i \text{ mod } 600$
15	681	$b(i)=81*i \text{ mod } 682$

### D(1,10)

k	Upper b.	Construction (pattern)
2	6	$b(i)=3i \text{ mod } 7$
3	13	0 8 2 10 4 12 6 0 8 3 11 5 13 7 1 9 3 11 5 13 7 2 9 4 12 6 0 8 2 10 4 12 6
4	26	$b(i)=7i \text{ mod } 27$
5	52	$b(i)=14i \text{ mod } 54$
6	70	$b(i)=31i \text{ mod } 71$
7	110	$b(i)=47i \text{ mod } 111$
8	150	$b(i)=12i \text{ mod } 151$

9	208	$b(i)=12i \pmod{209}$
10	276	$b(i)=78i \pmod{278}$
11	354	$b(i)=132i \pmod{357}$
12	429	$b(i)=159i \pmod{430}$
13	532	$b(i)=29i \pmod{533}$
14	632	$b(i)=34i \pmod{633}$
15	744	$b(i)=128i \pmod{748}$

**D(1,11)**

k	Upper b.	Construction (pattern)
2	6	$b(i)=3i \pmod{7}$
3	12	0 9 6 3 10 1 6 11 4 1 12 7 4 1 12 7 4 9 2 7 10 5 2 11 8 5 2 11 0 5 12 3 0 9 6 3
4	26	$b(i)=4i \pmod{27}$
5	37	$b(i)=11i \pmod{38}$
6	74	$b(i)=22i \pmod{76}$
7	97	$b(i)=43i \pmod{98}$
8	148	$b(i)=16i \pmod{150}$
9	177	$b(i)=19i \pmod{178}$
10	262	$b(i)=122i \pmod{263}$
11	329	$b(i)=17i \pmod{330}$
12	436	$b(i)=166i \pmod{437}$
13	513	$b(i)=113i \pmod{514}$
14	644	$b(i)=308i \pmod{648}$
15	747	$b(i)=27i \pmod{748}$

**D(1,12)**

k	Upper b.	Construction (pattern)
2	6	$b(i)=2i \pmod{7}$
3	13	0 5 10 3 8 0 6 11 3 9 5 2 12 8 1 6 12 4 9 1 7 12 0 10 6 3 13 9 2 7 13 5 10 2 8 13 1 11 7 0 5 11 3 8 0 6 11 3 9 5 2 12 8 1 6 12 4 9 1 7 12 0 10 6 3 13 9 2 7 13 5 10 2 8 13 1 11 7

4	28	0 8 13 18 25 1 6 13 18 23 28 6 11 16 21 28 4 9 16 21 26 8 3 14 19 24 2 7 12 19 24 0 5 12 17 22 27 5 10 15 22 27 3 10 15 20 25 1 8 13 18 25 1 6 13 18 23 28 6 11 16 21 28 4 9 16 21 26 8 2 14 19 24 0 7 12 19 24 0 5 12 17 22 27 5 10 15 22 27 3 10 15 20 25
5	52	$b(i)=14i \text{ mod } 53$
6	72	$b(i)=31i \text{ mod } 73$
7	107	$b(i)=41i \text{ mod } 108$
8	146	$b(i)=62i \text{ mod } 147$
9	200	$b(i)=26i \text{ mod } 202$
10	272	$b(i)=102i \text{ mod } 274$
11	360	$b(i)=126i \text{ mod } 361$
12	463	$b(i)=59i \text{ mod } 464$
13	564	$b(i)=194i \text{ mod } 565$
14	660	$b(i)=228i \text{ mod } 664$
15	804	$b(i)=92i \text{ mod } 808$

**D(1,13)**

k	Upper b.	Construction (pattern)
2	7	0 4 7 3 6 4 1 6 4 7 0 6 4 2 6 1 5 0 2 7 3 0 2 5 3 1 7
3	12	0 11 6 1 8 3 10 5 0 7 10 1 12 7 2 9 4 11 6 1 8 11 2 5 8 3 10 5 12 7 2 9 12 3 6 9 0 11 6 1 8 3 10 5 0 7 10 1 12 7 2 9 4 11 6 1 8 11 2 5 8 3 10 5 0 7 2 9 12 3 6 9
4	27	0 8 16 24 5 13 21 1 9 18 26 6 14 23 3 11 19 27 8 16 24 4 12 21 1 9 17 26 6 14 22 2 11 19 27 7 15 24 4 12 20 0 9 17 25 5 14 22 2 10 18 27 7 15 23 3 12 20 0 8 17 25 5 13 21 2 10 18 26 6 15 23 3 11 20
5	41	$b(i)=11i \text{ mod } 42$
6	78	$b(i)=15i \text{ mod } 79$
7	99	$b(i)=19i \text{ mod } 100$
8	160	$b(i)=76i \text{ mod } 162$
9	193	$b(i)=91i \text{ mod } 194$
10	278	$b(i)=17i \text{ mod } 279$
11	311	$b(i)=19i \text{ mod } 312$
12	426	$b(i)=195i \text{ mod } 427$
13	517	$b(i)=169i \text{ mod } 518$
14	660	$b(i)=180i \text{ mod } 664$
15	775	$b(i)=79i \text{ mod } 776$

**D(1,14)**

k	Upper b.	Construction (pattern)
2	7	$b(i)=3i \pmod 8$
3	12	0 8 2 10 5 12 7 2 9 4 12 6 1 9 3 11 6 0 8 3 10 5 0 7 2 10 4 12 7 1 9 4 11 6 1 8 3 11 5 0 8 2 10 5 12 7 2 9 4 12 6 1 9 3 11 6 0 8 3 10 5 0 7 2 10 4 12 7 1 9 4 11 6 1 8 3 11 5
4	27	0 19 11 3 22 14 6 25 17 9 0 20 12 4 23 15 7 26 18 1 11 21 4 14 24 7 17 27 10 2 21 13 5 24 16 8 27 19 11 3 22 14 6 25 17 0 10 20 3 13 23 6 16 26 9 1 20 12 4 23 15 7 26 18 10 2 21 13 5 24 16 8 27 19 2 12 22 5 15 25 8
5	48	$b(i)=11i \pmod{49}$
6	81	$b(i)=13i \pmod{82}$
7	108	$b(i)=19i \pmod{109}$
8	148	$b(i)=34i \pmod{150}$
9	212	$b(i)=44i \pmod{214}$
10	264	$b(i)=12i \pmod{267}$
11	344	$b(i)=67i \pmod{345}$
12	438	$b(i)=206 \pmod{439}$
13	554	$b(i)=44i \pmod{555}$
14	684	$b(i)=41i \pmod{685}$
15	820	$b(i)=396i \pmod{824}$

**D(1,15)**

k	Upper b.	Construction (pattern)
2	7	0 4 7 3 6 2 0 5 7 0 3 7 0 6 4 2 6 1 5 0 4 7 3 1 4 6 1 5 3 1 7
3	11	$b(i)=5i \pmod{12}$
4	27	0 5 11 16 21 26 3 8 13 18 23 0 5 10 15 20 25 2 8 13 18 23 0 5 10 15 20 25 2 7 12 17 22 27 5 10 15 20 25 2 7 12 17 22 27 4 9 14 19 24 1 7 12 17 22 27 4 9 14 19 24 1 6 11 16 21 27 3 9 14 19 24 0 6 11 16 21 26 3 8 13 18 23
5	48	$b(i)=8i \pmod{49}$
6	78	$b(i)=32i \pmod{79}$
7	103	$b(i)=17i \pmod{104}$
8	150	$b(i)=31i \pmod{151}$
9	185	$b(i)=83i \pmod{186}$
10	282	$b(i)=12i \pmod{285}$

11	331	$b(i)=27i \pmod{332}$
12	438	$b(i)=183i \pmod{441}$
13	493	$b(i)=205i \pmod{494}$
14	654-	$b(i)=37i \pmod{655}$
15	775	$b(i)=57i \pmod{776}$

## Lower bounds for $D(1,t)$ : (SAT solver)

k=2, t=10: 21 nodes graph was UNSAT using labels 0-5 (<1 second, 10 threads)  
k=2, t=11: 23 nodes graph was UNSAT using labels 0-5 (<1 second, 10 threads)  
k=2, t=12: 23 nodes graph was UNSAT using labels 0-5 (<1 second, 10 threads)  
k=2, t=13: 44 nodes graph was UNSAT using labels 0-6 (<1 second, 10 threads)  
k=2, t=14: 48 nodes graph was UNSAT using labels 0-6 (<1 second, 10 threads)  
k=2, t=15: 58 nodes graph was UNSAT using labels 0-6 (<1 second, 10 threads)

k=3, t=10: 34 nodes graph was UNSAT using labels 0-12 (<6 seconds, 10 threads)  
k=3, t=11: 35 nodes graph was UNSAT using labels 0-11 (<1 second, 10 threads)  
k=3, t=12: 37 nodes graph was UNSAT using labels 0-12 (25 seconds, 10 threads)  
k=3, t=13: 43 nodes graph was UNSAT using labels 0-11 (<1 second, 10 threads)  
k=3, t=14: 38 nodes graph was UNSAT using labels 0-11 (<1 second, 10 threads)  
k=3, t=15: 33 nodes graph was UNSAT using labels 0-10 (<1 second, 10 threads)

k=4, t=9: 37 nodes graph was UNSAT using labels 0-26 (10 threads, 103560 seconds)  
k=4, t=10: 33 nodes graph was UNSAT using labels 0-25 (10 threads, 600 seconds)  
k=4, t=11: 38 nodes graph was UNSAT using labels 0-25 (10 threads, 676 seconds)  
k=4, t=12: 50 nodes graph was UNSAT using labels 0-26 (10 threads, 3636 seconds)  
k=4, t=13: 110 nodes graph was UNSAT using labels 0-26 (10 threads, 5671 seconds)  
k=4, t=14: 58 nodes graph was UNSAT using labels 0-26 (10 threads, 4170 seconds)  
k=4, t=15: 64 nodes graph was UNSAT using labels 0-26 (10 threads, 10234 seconds)

k=5, t=10-15: 50 nodes graph was UNSAT using labels 0-29 (less than 10 minutes, 10 threads)  
k=6, t=10-15: 50 nodes graph was UNSAT using labels 0-39 (less than 10 minutes, 10 threads)  
k=7, t=12-15: 50 nodes graph was UNSAT using labels 0-44 (less than 10 minutes, 10 threads)