VAJE 2.: FUNKCIJE VEČ SPREMENLJIVK

NIVOJNICE IN DEFINICIJSKO OBMOČJE

(1.) Poišči in skiciraj definicijsko območje funkcij

$$f(x,y) = \frac{\sqrt{4x-y^2}}{\ln(1-x^2-y^2)}$$ in

$$g(x,y) = \ln(x\ln(y-x)).$$

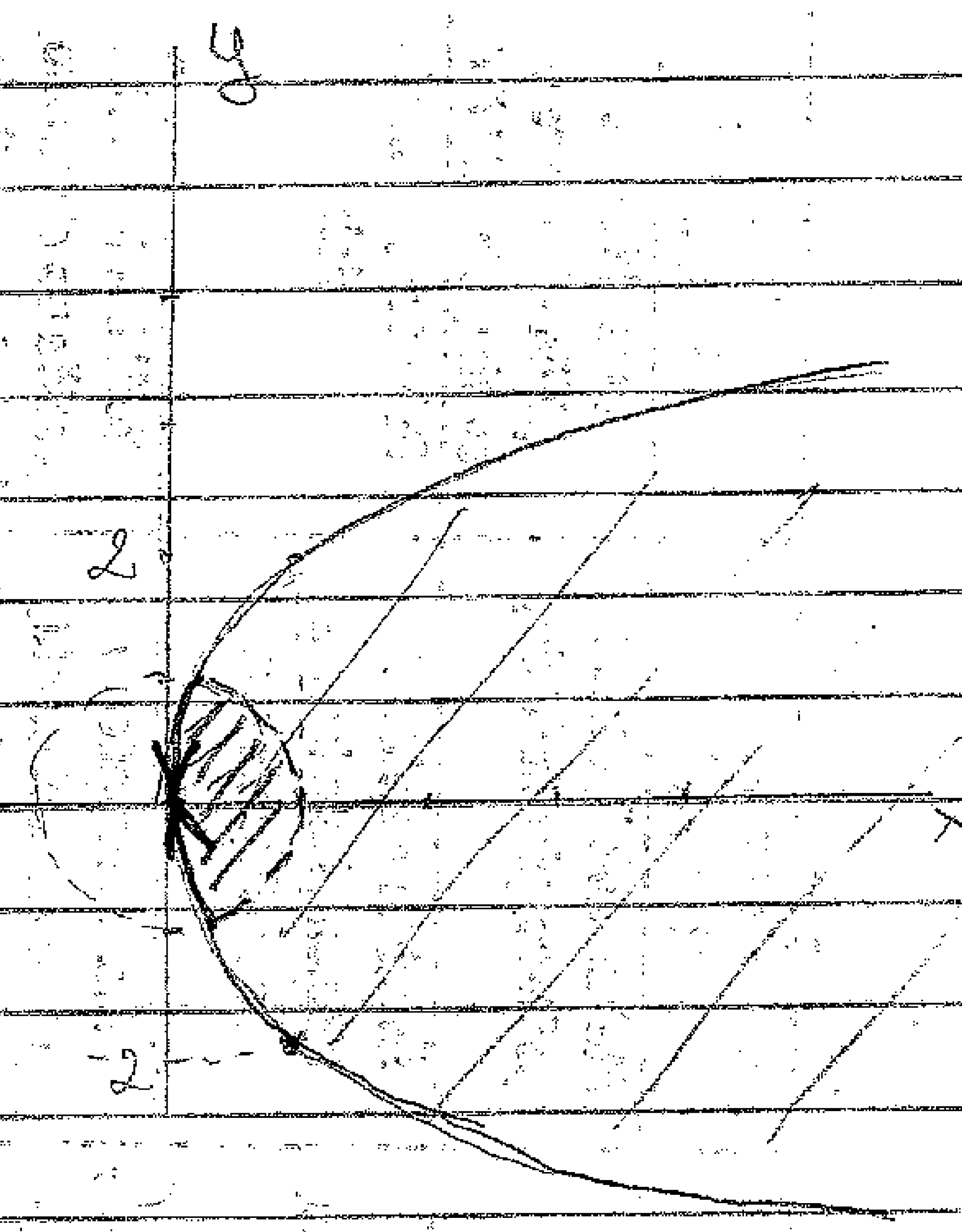
a) $$f(x,y) = \frac{\sqrt{4x-y^2}}{\ln(1-x^2-y^2)}$$

$f: \mathbb{R}^2 \to \mathbb{R}$

D_f : $4x-y^2 \geq 0$ & $\ln(1-x^2-y^2) \neq 0$ & $1-x^2-y^2 >$

$-y^2 \geq -4x$

$y^2 \leq 4x$



$y^2 \leq 4x$

$y^2 - 4x = 0$

parabola

$\to x = \frac{1}{4}y^2$

$y^2 \leq 4x \quad / \sqrt{\ }$

$\sqrt{y^2} \leq 2\sqrt{x}$

$|y| \leq 2\sqrt{x}$

$\ln(1 - x^2 - y^2) \neq 0 \Rightarrow 1 - x^2 - y^2 \neq 1$

$x^2 + y^2 \neq 0$

$x^2 + y^2 = 0 \Rightarrow x, y = 0$ tzn. $x, y \neq 0$

$1 - x^2 - y^2 > 0$

$-x^2 - y^2 > -1$

$x^2 + y^2 < 1$

b) $g(x,y) = \ln(x \cdot \ln(y-x))$

1.) $x \cdot \ln(y-x) > 0$ 2.) $y - x > 0$

I. $x > 0$ & $\ln(y-x) > 0$

$y - x > 1$

$y > 1 + x$

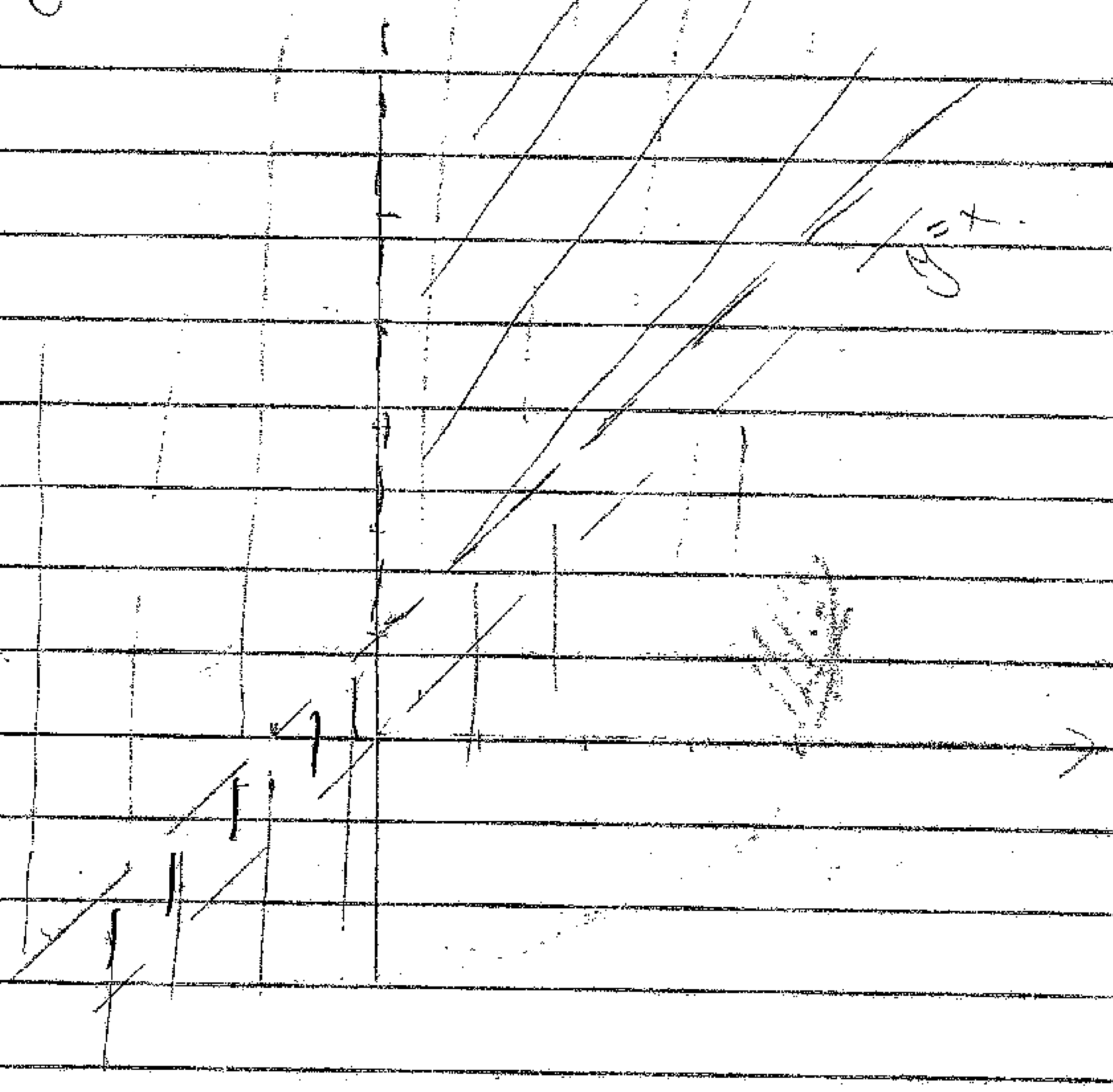
II. $x < 0$ & $\ln(y-x) < 0$

$\ln(y-x) < 0$

$\Rightarrow 0 < y - x < 1$

$x < y < x + 1$

1.1. $x>0$ & $y>1+x$



1.2. $x<y<x+1$

(2) S pomočjo nivojnic skiciraj grafa funkcij:

$f(x,y)=\frac{1}{x^2+y^2}$ in $g(x)=\frac{x}{x^2+y^2}$

$f:\mathbb{R}^2\to\mathbb{R}$

$\Gamma_f=\{(x,y,z) \mid z=f(x,y),\ (x,y)\in D_f\}\subseteq\mathbb{R}^3$

↳ predstavlja ploskev

$z=f(x,y)$

nivoj. fun

$N_a=\{(x,y)\in\mathbb{R}^2;\ f(x,y)=a\}=f^{-1}(\{a\})$

nivojnica

$a\in\mathbb{R}$

praslika

a) $f(x,y) = \dfrac{1}{x^2+y^2}$

$D_f: x^2+y^2 \neq 0$

$D_f = \mathbb{R}^2 \setminus \{(0,0)\}$

$Z_f: \mathbb{R}^+$ (zaradi x^2 in y^2)

$N_1: \dfrac{1}{x^2+y^2} = 1 \Rightarrow x^2+y^2 = 1$

↳ krožnica $K_1(0,0)$

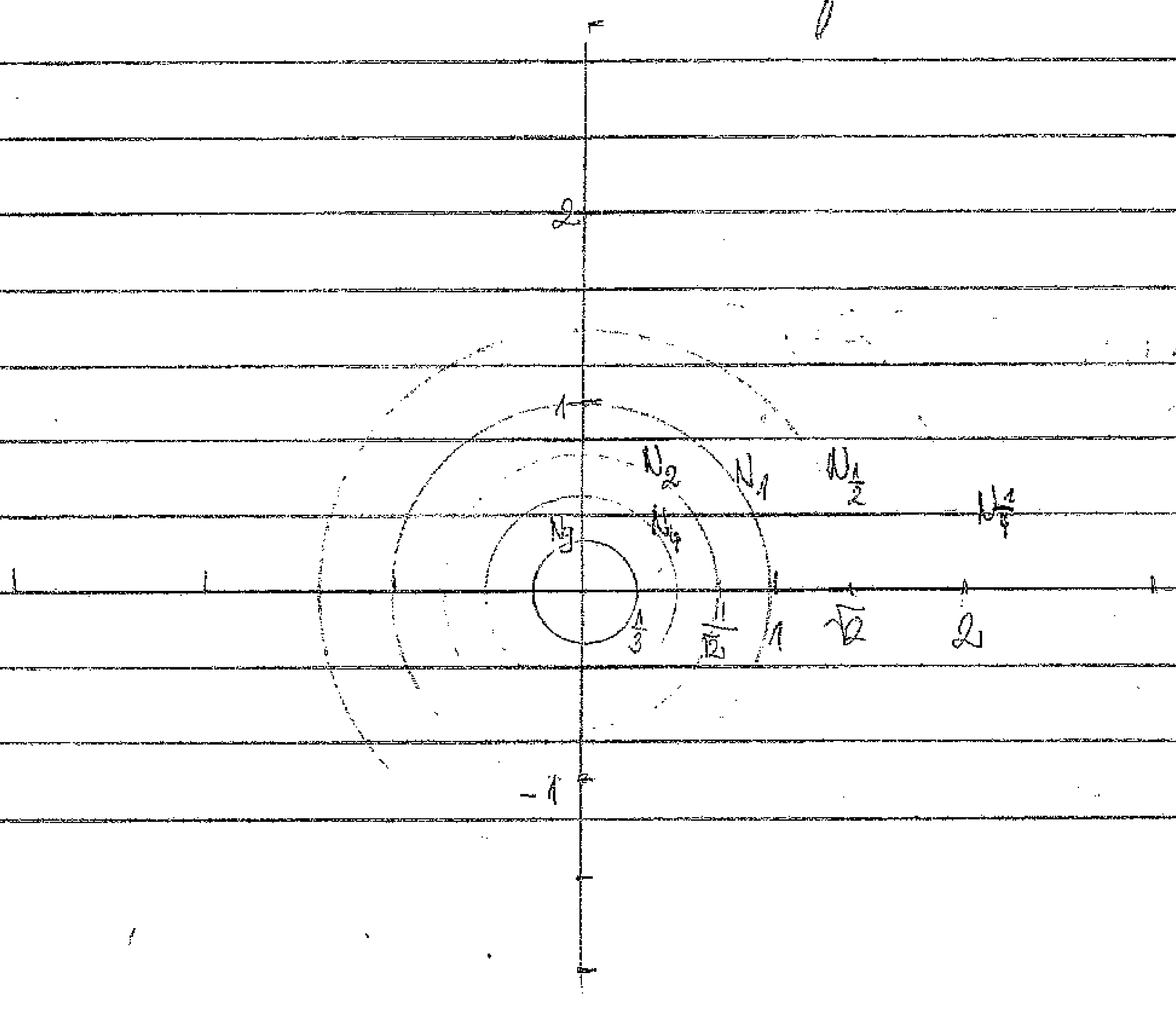
$N_2: \dfrac{1}{x^2+y^2} = 2 \Rightarrow x^2+y^2 = \dfrac{1}{2} \Rightarrow K_{\frac{1}{\sqrt{2}}}(0,0)$

$N_4: \dfrac{1}{x^2+y^2} = 4 \Rightarrow x^2+y^2 = \dfrac{1}{4} \Rightarrow K_{\frac{1}{2}}(0,0)$

$N_{\frac{1}{4}}: \dfrac{1}{x^2+y^2} = \dfrac{1}{4} \Rightarrow x^2+y^2 = 4 \Rightarrow K_2(0,0)$

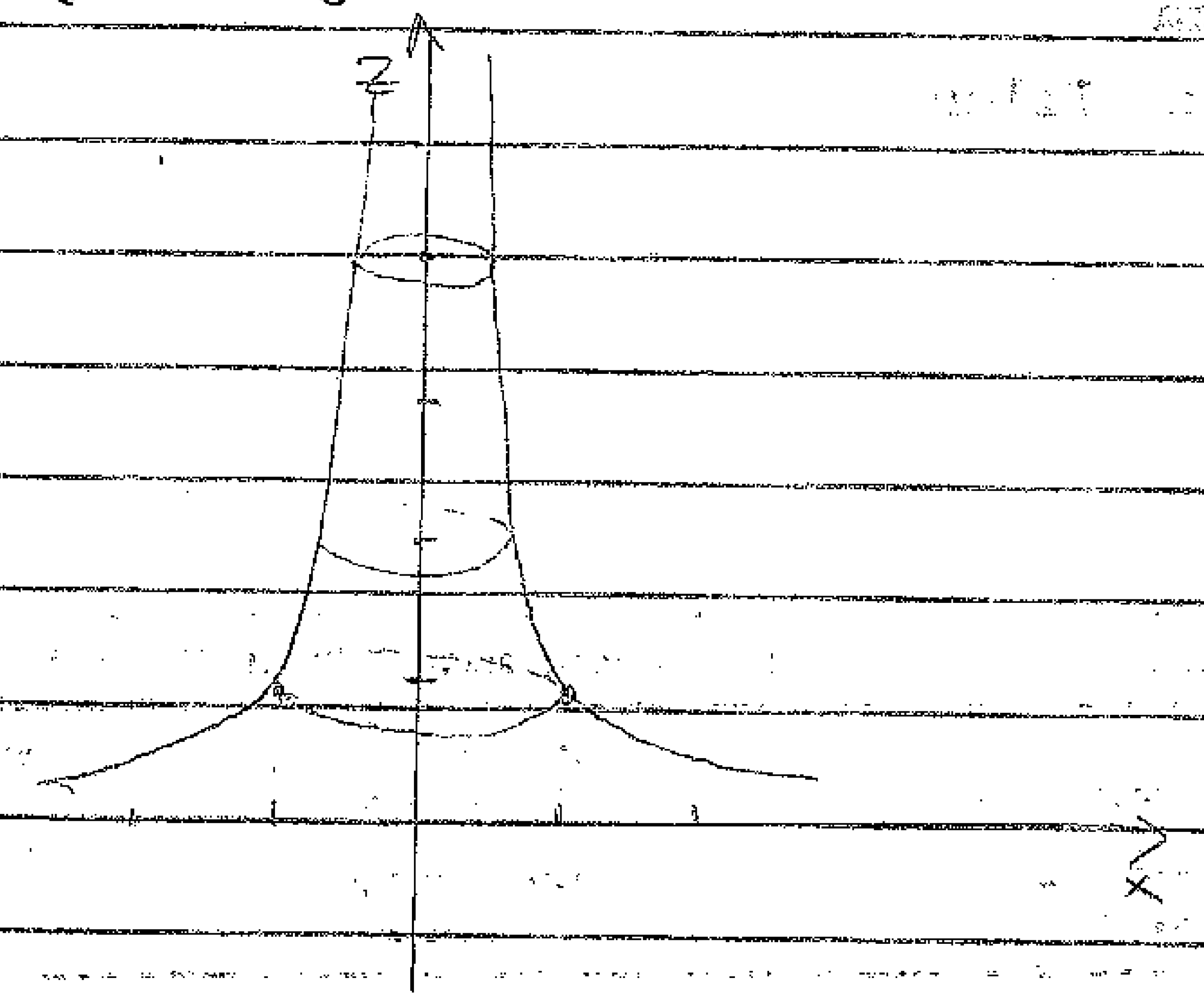
$a \in (0,\infty)$

$N_a: K_{\frac{1}{\sqrt{a}}}(0,0)$ splošno zapisana nivojnica.



Prerez grafa ~~mimo~~ grafa nad osjo x

$z = f(x,y) = f(x,0) = \frac{1}{x^2}$

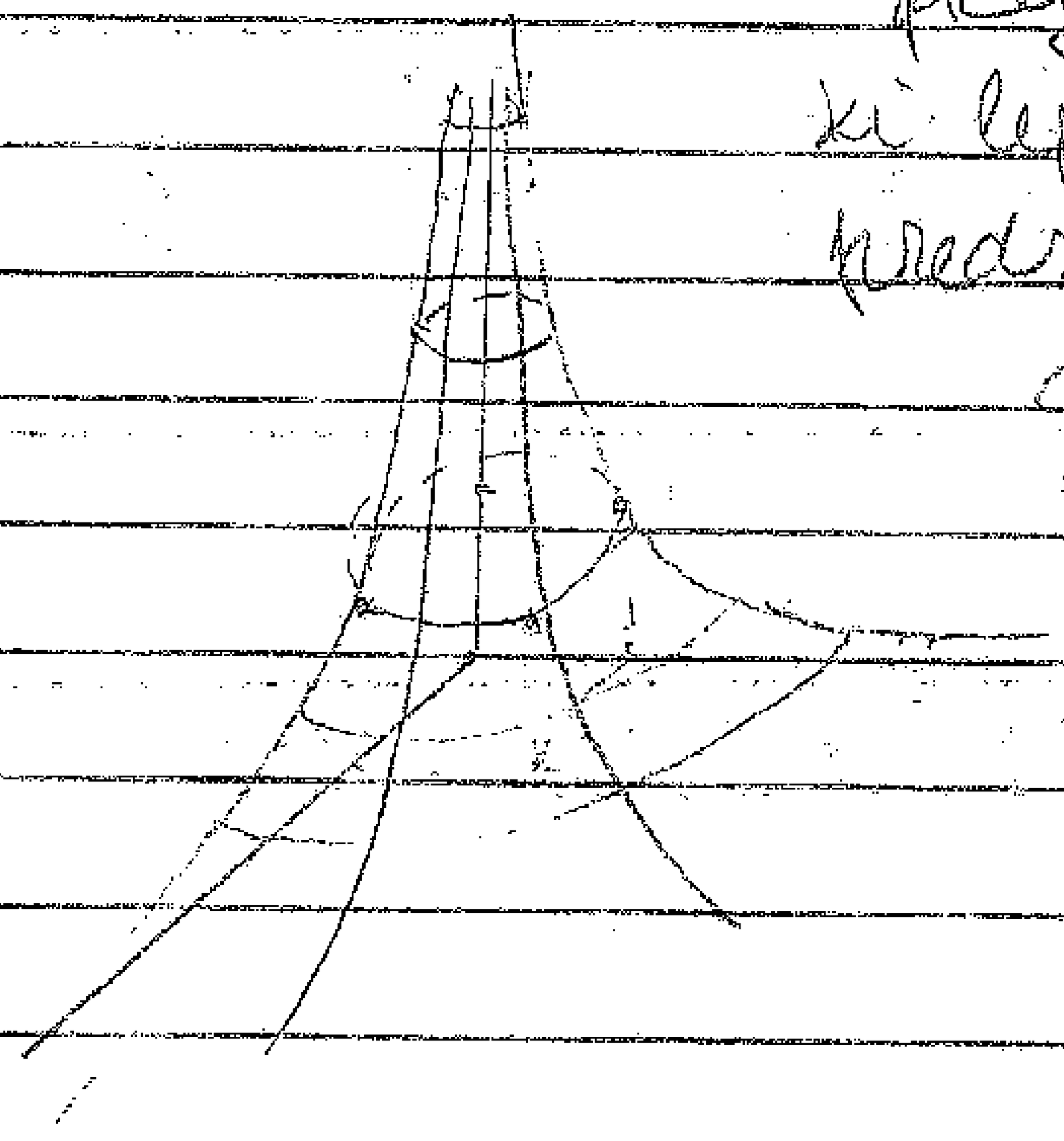


DPGRAPH
↳ program ki lepo predstavi grafe

$x = r \cdot \cos \varphi$

$y = r \cdot \sin \varphi$

$f(x,y) = \frac{1}{r^2}$



b) $g(x,y) = \frac{x}{x^2+y^2}$

$D_g: \mathbb{R}^2 \setminus \{(0,0)\}$

$Z_g: \mathbb{R}$

$N_1: \frac{x}{x^2+y^2} = 1$

$x^2+y^2 = x \Rightarrow x^2 - x + y^2 = 0 \Rightarrow (x-\frac{1}{2})^2 - \frac{1}{4} + y$

$S(\frac{1}{2},0) \quad r=\frac{1}{2} \Rightarrow K_{\frac{1}{2}}(\frac{1}{2},0)$

$N_0: \frac{x}{x^2+y^2}=0 \Rightarrow x=0$

$N_{-1}: \frac{x}{x^2+y^2}=-1 \quad / x^2+y^2$

$\frac{1}{2}\cdot\frac{1}{2}=1$

$\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$

$x=-x^2-y^2$

x^2+x+y^2

$(x+\frac{1}{2})^2+y^2=\frac{1}{4} \Rightarrow K_{\frac{1}{2}}(-\frac{1}{2},0)$

$N_a: \frac{x}{x^2+y^2}=a \quad / x^2+y^2$

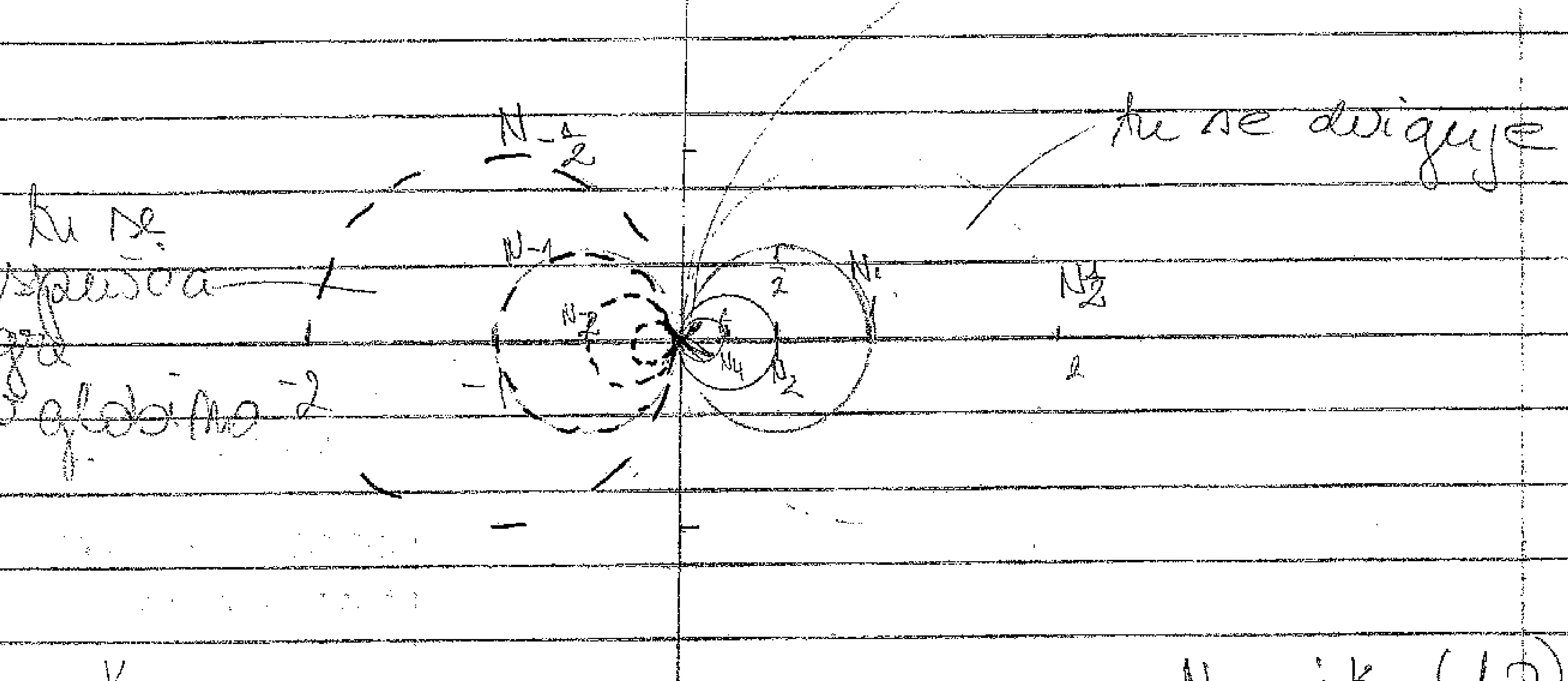
$x=a(x^2+y^2)$

$\frac{x}{a}=(x^2+y^2)$

$x^2-\frac{x}{a}+y^2=0$

$(x-\frac{1}{2a})^2+y^2=\frac{1}{4a^2}$ $\Rightarrow \sqrt{4a^2}=2|a|$

$K_{\frac{1}{2|a|}}(\frac{1}{2a},0)$



$N_{\frac{1}{2}}: K$

$N_2: K_{\frac{1}{4}}(\frac{1}{4},0)$

$N_{-\frac{1}{2}}: K_1(-1,0)$

glede na x je funkcija liha

$$z = g(x,y) = \frac{x}{x^2+y^2}$$

$$g(-x,y) = -g(x,y)$$

glede na y

$g(x,-y) = g(x,y)$ je soda

Prerežimo jo pri $y=0$

$$z = g(x,0) = \frac{x}{x^2} = \frac{1}{x}$$