Essential Lower Bounds on the Matching Number and Essential Upper Bounds on the Total Domination Number

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Abstract

The matching number, $\alpha'(G)$, of a graph G is the number of edges in a maximum matching of G. The total domination number, $\gamma_t(G)$, is the minimum cardinality of a total dominating set of G, where a set S of vertices in a graph G is a total dominating set of G if every vertex has a neighbor in S. Let n_G and m_G denote the number of vertices and edges, respectively, in G.

The first result in this talk is to prove that all essential lower bounds on the matching number of a graph with maximum degree k can be written in a unified form for all $k \geq 3$. For this purpose, we give a complete description of the set L_k of pairs (γ, β) of real numbers with the following property. There exists a constant K such that $\alpha'(G) \geq \gamma n_G + \beta m_G - K$ for every connected graph G with maximum degree at most k.

Our second result is to prove that all the essential upper bounds on the total domination number of a graph G without isolated vertices and isolated edges can be written in the unified form $\gamma_t(G) \leq (2an_G + 2bm_G)/(3a + 2b)$ for constants $b \geq 0$ and $a \geq \frac{2}{3}(1 - b)$.

References

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