# Symmetry Breaking in Graphs and 1-2-3 Conjecture

Monika Pilśniak
Department of Discrete Mathematics, AGH University, Krakow, Poland
pilsniak@agh.edu.pl

A colouring of a graph $G$ is called *asymmetric* if the identity is the only automorphism preserving the colouring. The *distinguishing index* $D'(G)$ of a graph is the least number of colours in an asymmetric edge colouring. It was defined by Kalinowski and Pilśniak in [1].

In the talk, we survey results on asymmetric edge colourings of finite graphs. We give known general upper bounds in terms of maximum degree. We focus mainly on several classes of graphs which need only two or three colours to break all nontrivial automorphisms [4].

On the other hand, we say that a colouring $c : E(G) \longrightarrow \{1, \ldots, k\}$ is *neighbour-distinguishing by sums* if

$$\sum_{e \ni u} c(e) \neq \sum_{e \ni v} c(e), \qquad uv \in E(G).$$

Karoński, Łuczak and Thomason [3] formulated the 1-2-3 Conjecture that every connected graph of order $n \geq 3$ admits such a colouring with $k = 3$. This conjecture has been confirmed for some classes of graphs, but in general it remains open since 2004. Up to now, the best result for $k = 5$ is due to Kalkowski, Karoński and Pfender.

In this talk, we introduce a class of automorphisms such that edge colourings breaking them are connected to edge colourings distinguishing neighbours by sums. We call an automorphism $\varphi$ of a graph $G$ *small* if there exists a vertex of $G$ that is mapped by $\varphi$ onto its neighbour. The *small distinguishing index* of $G$, denoted $D_s(G)$, is the least $k$ such that there exists a $c$ breaking all small automorphisms of $G$. We prove that $D_s(G) \leq 3$ for every connected graph $G$ of order $n \geq 3$, thus supporting, in a sense, the 1-2-3 Conjecture [2].

We also study an analogous problem for total colourings in connection with the 1-2 Conjecture of Przybyło and Woźniak.

[1] R. Kalinowski, M. Pilśniak, *Distinguishing graphs by edge-colourings*, European J. Combin. 45 (2015), 124–131.
[2] R. Kalinowski, M. Pilśniak, M. Woźniak, *A note on Breaking Small Automorphisms in Graphs*, Discrete Appl. Math. 232 (2017), 221–225.
[3] M. Karoński, T. Łuczak, A. Thomason, *Edge weights and vertex colours*, J. Combin. Theory Ser. B 91 (2004), 151–157.
[4] M. Pilśniak, *Improving upper bounds for the distinguishing index*, Ars Math. Contemp. 13 (2017), 259–274.