Super-connectivity of graphs

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Classical connectivity studies the minimum number of vertices or edges that need to be deleted in order to disconnect a graph, without putting any restrictions on the resulting components. In 1983, Harary introduced the notion of conditional connectivity which asks for the minimum number of vertices or edges that need to be deleted such that the graph is disconnected and every resulting component exhibits a certain prescribed property. Although, theoretically, any property $\mathcal{P}$ can be chosen, the most common choices of $\mathcal{P}$ are those that have a practical application.

In this talk we restrict our attention to the super-connectivity of a graph. This notion studies the least number of vertices that need to be deleted from a graph to disconnect the graph such that each remaining component is not trivial. In other words, the property $\mathcal{P}$ that each component must possess is that of *not* being an isolated vertex, or equivalently, that of containing at least one edge. The reason behind this choice of $\mathcal{P}$ lies primarily in the study of interconnection networks. Network design focuses on determining how to best connect nodes such that the resulting network is, among other considerations, efficient, resistant to threats, cost-effective and fault tolerant. It has been argued by many that in large–scale processing systems, the possibility of all the vertices adjacent to a single vertex of the graph fail simultaneously, hence creating an isolated vertex, is particularly remote. Thus, connectivity parameters which preclude that none of the components of the resulting graph is an isolated vertex are, generally, more desirable and provide better measures of the reliability of a network. Consequently, in such instances, the super–connectivity of a graph, if it exists, provides a better measure of reliability.

We will present well-known bounds for the super–connectivity of graphs and discuss some conditions for the bounds to be attained. Finally, we review the super-connectivity of some families of graphs, including circulant graphs, hypercubes, generalized Petersen graphs, Kneser graphs and Johnson graphs.