On several mathematical aspects of Wiener index

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Abstract. The Wiener index $W(G)$ of a simple connected graph G is defined as the sum of distances over all pairs of vertices in G. The index is of interest in chemistry as it correlates well with chemical properties of some molecules, but it also has interesting mathematical aspects. A survey of mathematical results and open problems regarding the Wiener index is given in [1]. We are going to consider two conjectures from [1], and present a solution to them. For a class $\mathcal{T}_n$ of trees on n vertices, $W[\mathcal{T}_n]$ denotes the set of values of Wiener index on trees from $\mathcal{T}_n$. The first conjecture states that the cardinality of the largest interval of contiguous integers (contiguous even integers in case of odd n) contained in $W[\mathcal{T}_n]$ is $\frac{1}{6}n^3 + O(n^2)$ in case of even n, and $\frac{1}{12}n^3 + O(n^2)$ in case of odd n. The line graph $L(G)$ of a graph G is defined as a graph having a vertex set identical with the set of edges of G and two vertices of $L(G)$ are adjacent if and only if the corresponding edges are incident in G. The second conjecture states that the ratio $W(L(G))/W(G)$ attains maximum if and only if G is a complete graph K_n. We show that both conjectures are true.

[1] M. Knor, R. Škrekovski, A. Tepeh, Mathematical aspects of Wiener index, *Ars Math. Contemp.* **11** (2016) 327–352.