Optimal linear-Vizing relationships for (total) domination in graphs

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In this talk, we discuss the following two problems, where $\gamma(G)$ and $\gamma_t(G)$ denote the domination and total domination numbers, respectively, of a graph G.

Problem 1. *For each $\Delta \geq 3$, find the smallest value, c_Δ, such that for every connected graph G of order n, size m, domination number $\gamma(G) = \gamma$, and bounded maximum degree $\Delta(G) \leq \Delta$,*

$$m \leq \left(\frac{\Delta + c_\Delta}{2}\right) n - \left(\frac{\Delta + c_\Delta + 2}{2}\right) \gamma.$$

Problem 2. *For each $\Delta \geq 3$, find the smallest value, r_Δ, such that for every connected graph G of order $n \geq 3$, size m, total domination number γ_t, and bounded maximum degree $\Delta(G) \leq \Delta$,*

$$m \leq \frac{1}{2}(\Delta + r_\Delta)(n - \gamma_t).$$

For all $\Delta \geq 3$, Rautenbach [3] in 1999 showed that $c_\Delta \leq \Delta$. For all $\Delta \geq 3$, Shan, Kang, and Henning [1, 4] in 2005 showed that $r_\Delta \leq \Delta$. Subsequently, Yeo [5] in 2007 showed that $0.05 \ln(\Delta) < c_\Delta$ and $0.1 \ln(\Delta) < r_\Delta \leq 2\sqrt{\Delta}$ for all $\Delta \geq 3$, and posed as an open problem to determine "whether r_Δ grows proportionally with $\ln(\Delta)$ or $\sqrt{\Delta}$ or some completely different function." In this talk, we determine the growth of r_Δ, and show that both c_Δ and r_Δ are asymptotically $\ln(\Delta)$. This talk is based on joint work with Paul Horn.

[1] M. A. Henning, A linear Vizing-like relation relating the size and total domination number of a graph. *J. Graph Theory* **49** (2005), 285–290.

[2] M. A. Henning and P. Horn, Optimal linear-Vizing relationships for (total) domination in graphs, manuscript.

[3] D. Rautenbach, A linear Vizing-like relation between the size and the domination number of a graph. *J. Graph Theory* **31** (1999), 297–302.

[4] E. Shan, L. Kang, and M. A. Henning, Erratum to: A linear Vizing-like relation relating the size and total domination number of a graph. *J. Graph Theory* **54** (2007), 350–353.

[5] A. Yeo, Relationships between total domination, order, size, and maximum degree of graphs. *J. Graph Theory* **55** (2007), 325–337.