The 1st Maribor Mini Workshop on

Topological Dynamics, Continuum Theory, and related topics

Faculty of Natural Sciences and Mathematics
University of Maribor
Slovenia

18 - 19 January 2024

Welcome to the 1st Maribor Workshop on Topological Dynamics, Continuum Theory and Related Topics. This workshop (which we hope is the 1st in many) aims to bring together experts as well as young researchers working on the above mentioned topics to discuss and share contemporary ideas as well as establish new and strengthen the existing collaborations.

Figure 1: University of Maribor

All talks and collaboration activities will take place in the lecture room 0/103 at the Faculty of Natural Sciences and Mathematics, Koroška cesta 160, 2000 Maribor. The conference dinner will take place on Thursday Jan 18 at 19.00 at the restaurant Ancora in Maribor city center. On Friday Jan 19 at 18.30 we will have an excursion to Vinag wine cellars.

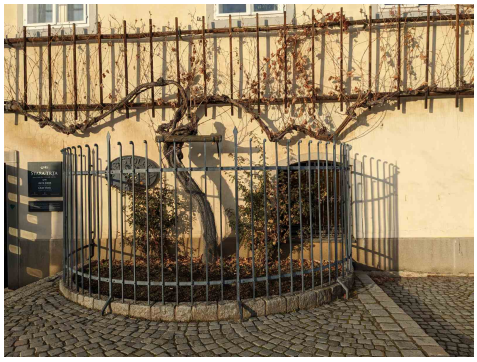

Figure 2: Old Vine

We wish you an enjoyable and fruitful workshop!

Organizers:
Iztok Banič
Jernej Činč
Matevž Črepnjak
Teja Kac
Tina Sovič

The conference is supported by Slovenian Research and Innovation Agency (ARIS) project J1-4632.

Schedule

All talks are in the room 0/103.

Thursday Jan 18 2024

8:30 – 9:00	Coffee
9:00 – 9:40	Piotr Oprocha
9:45 – 10:25	Jan P. Boroński
10:30 – 11:10	Sonja Štimac
11:15 – 13:30	Discussion and lunch
13:30 – 14:10	Udayan B. Darji
14:15 – 14:55	Judy Kennedy
15:00 – 15:30	Coffee
15:30 – 16:10	Van Nall
16:15 – 16:55	Nikola Koceić-Bilan
17:00 – 17:25	Michał Kowalewski
17:30 – 17:55	Jakub Tomaszewski

Friday Jan 19 2024

8:30 – 9:00	Coffee
9:00 – 9:40	Chris Mouron
9:45 – 10:25	Lori Alvin
10:30 – 11:10	Stefano Luzzatto
11:15 – 11:55	Maik Gröger
12:00 – 13:30	Lunch
13:30 – 14:10	Goran Erceg
14:15 – 14:55	Ivan Jelić
15:00 – 15:25	Rene Gril Rogina
15:30 – 15:55	Domagoj Jelić
16:00 – 16:30	Coffee

Contents

Schedule **iii**

Abstracts **1**

On tracing properties, invariant measures, and entropy (*Piotr Oprocha*) . . 1
The pruning front conjecture, folding patterns and classification of Hénon maps in the presence of strange attractors, part I (*Jan P. Boroński*) . 1
The pruning front conjecture, folding patterns and classification of Hénon maps in the presence of strange attractors, part II (*Sonja Štimac*) . . 2
Continua, groups and solenoids (*Udayan B. Darji*) 2
Dynamics admitted by the Lelek Fan (*Judy Kennedy*) 2
Shift maps on Mahavier products with closed relations (*Van Nall*) 2
A topological point of view to differentiability (*Nikola Koceić-Bilan*) 3
Topological properties of trenched graphs (*Michał Kowalewski*) 4
On extending Cantor subsystems on dendrites (*Jakub Tomaszewski*) 4
Strongly commuting maps (*Chris Mouron*) 4
Unimodal Maps, Substitutions, and Toeplitz Sequences (*Lori Alvin*) 5
Statistical and Non-Statistical Dynamics (*Stefano Luzzatto*) 5
Long-range order, mean equicontinuity and amorphic complexity (*Maik Gröger*) 5
Quotient of dynamical systems (*Goran Erceg*) 6
A note on finite coarse shape groups (*Ivan Jelić*) 6
End-point-generated smooth fans (*Rene Gril Rogina*) 6
On limit sets in hyperspace of continua in dimension one (*Domagoj Jelić*) . 7

Author Index **9**

Abstracts

On tracing properties, invariant measures, and entropy

Thu 9:00

Piotr Oprocha
AGH Krakow

In 1970s Bowen related hyperbolic dynamics with specification property and used this to show that there exists a unique measure of maximal entropy. Almost the same time Sigmund used specification property as a tool in characterization of simplex of invariant measures. Since then, these results were inspiration for numerous mathematicians in various studies of dynamics. Several weaker versions of specification property were developed and used as a tool for better understanding of dynamics. At the same time, questions, how often such properties can be found in dynamics were raised (e.g. in the sense of Baire category theorem). In this talk we will present selected questions and results fitting into the above framework of research.

The pruning front conjecture, folding patterns and classification of Hénon maps in the presence of strange attractors, part I

Thu 9:45

Jan P. Boroński
Jagiellonian University

In my talk I shall discuss my work with Sonja Štimac on Henon maps with strange attractors (Wang-Young parameters). I shall explain a construction (inspired by a work of Crovisier and Pujals) of conjugacy of these maps to the shift homeomorphisms on inverse limits of dendrites with dense set of branch points, and a characterization of orbits of critical points in terms of these inverse limits.

Thu 10:30

The pruning front conjecture, folding patterns and classification of Hénon maps in the presence of strange attractors, part II

Sonja Štimac
University of Zagreb

In my talk, I will discuss my work with Jan Boronski on Henon maps with strange attractors. I will explain our proof of the pruning front conjecture and folding patterns. I will also present how we classified (up to conjugacy) the Henon maps for the Wang-Young parameters combining the inverse limit approach and the symbolic dynamics.

Thu 13:30

Continua, groups and solenoids

Udayan B. Darji
University of Louisville

If we choose a compact, connected, metric group at random, what does it look like? We discuss the setting of this question, give a full answer and raise some open questions. Thisis joint work with Márton Elekes, Tamás Kátay, Anett Kocsis, and Máté Pálfy.

Thu 14:15

Dynamics admitted by the Lelek Fan

Judy Kennedy
Lamar University

We discuss recent work investigating the topological dynamics admitted by the Lelek fan. This is joint work with Iztok Banič, Goran Erceg, Chris Mouron, and Van Nall.

Thu 15:30

Shift maps on Mahavier products with closed relations

Van Nall
University of Richmond

We construct continua and functions at the same time that have a variety of dynamical properties. The continua are Mahavier products also known as an inverse limit with a set valued function. The functions are shift maps. The dynamics of shift maps on inverse limits is an old topic, but we will extend to continua that cannot be expressed as an inverse limit with a single continuous function. Specifically we look at various ways to express Cantor fans and the Lelek fan as Mahavier products. We obtain transitive homeomorphisms, mixing homeomorphisms, with and without a dense set of periodic orbits. This is joint work with Iztok Banič, Judy Kennnedy, Chris Mouron, and Goran Erceg.

A topological point of view to differentiability

Thu 16:15

Nikola Koceić-Bilan
University of Split

The differentiability of scalar and vector functions of multiple variables is defined only at the interior points of the domain of these functions, which results in the traditional consideration of functions only with an open domain in $\mathbb{R}^n$. This significantly narrows the possibility of applying potential techniques and tools of differential calculus to a wider class of functions. Although there is a strong need for it in various problems of mathematical analysis and other mathematical branches, so far, the notion of differentiability of a function has not been considered or successfully deÖned at points outside the interior of domain of a function. In this talk, we will define the differentiability at all points of a domain $X \subseteq \mathbb{R}^n$ of a function $f : X \to \mathbb{R}^m$ in which that notion makes sense. These are the points that admit neighbourhood ray in X which is the minimum condition for the notion of linearization of a function (the essential property of differentiable functions) to make sense. In such a way, the notion of differentiability is significantly expanded, leading to a new theory of differentiable functions that offers completely unexpected phenomena and pathologies (such as the non-uniqueness of differentials, the discontinuity of differentiable functions...), but also reveals some common misconceptions. However, if one reduces this theory only to the points with particularly special properties (points that admit raylike neighbourhood and a linearization space with dimensions equal to the dimension of the Euclidean space to which the domain belongs), then all properties and assertions of the extended theory remain the same. Moreover, all known theorems and techniques of the differential calculus can be successfully generalized and support the new theory, whereby the derivatives in the direction of the chosen vectors take over the role of partial derivatives. This is especially important for the functions which are differentiable at the point where there are no partial derivatives of them. If $P \in X \subseteq \mathbb{R}^n$ admits neighbourhood ray in X in the direction of some n linear independent vectors in $\mathbb{R}^n$ we will investigate under which conditions the existence of derivatives in the direction of those vectors at the point P implies the differentiability of a function $f : X \to \mathbb{R}^m$ at P.

References:

- N.K.B., S. Braić, Generalized Approach to Differentiability, Mathematics 2022, 10(17),
- N.K.B., S. Braić, Continuous Differentiability in the Context of Generalized Approach to Differentiability, Mathematics 2023, 11 (6), 1445
- N.K.B., I. Jelić, The Mean Value Theorem in the Context of Generalized Approach to Differentiability, Mathematics 2023, 11, 4294

Thu 17:00

Topological properties of trenched graphs

Michał Kowalewski
AGH Krakow

Topological closure of the graph of the function $\sin(1/x)$ defined on interval $(0, 1]$ is a standard example of a connected space that is not arcwise connected. By adding an arc to this continuum, we get a Warsaw circle - arcwise connected, but not locally arcwise connected continuum. In this talk I will present topological properties of two classes of continua that are inspired by concepts underlying the Warsaw circle-quasi-graphs and $\sin(1/x)$-type continua. Family of trenched graphs encompasses both of these classes. We will discuss similarities and surprising differences between quasi-graphs and $\sin(1/x)$-type continua, as well as provide characterizations that link them together.

Thu 17:30

On extending Cantor subsystems on dendrites

Jakub Tomaszewski
AGH Krakow

During the talk we will focus on surjective Cantor systems. Each such system can be easily embedded in the Gehman dendrite, as its set of endpoints is a Cantor set. We will show that for each such embedding there exists a mixing map of the dendrite such that the endpoints' subsystem is conjugate to the Cantor system of choice. The main tool to obtain this result follows from Shimomura's method of approximating the dynamics on zero dimensional systems by analysing the dynamics of coverings of the underlying space. We will discuss the dynamical properties of the constructed map.

The talk is based on joint work with Dominik Kwietniak and Piotr Oprocha.

Fri 9:00

Strongly commuting maps

Chris Mouron
Rhodes College

Let $f, g : X \longrightarrow X$. We say that f and g *commute* if $f(g(x)) = g(f(x))$ for all $x \in X$. Maps f, g that strongly commute when $f^{-1} \circ g = g \circ f^{-1}$. In this talk, I will discuss questions and solutions about strongly commuting maps of the particular spaces X. From here, I will discuss applications of this to entropy and also to fixed point theory.

This is joint work with Ana Anušič and Anna Cole.

Unimodal Maps, Substitutions, and Toeplitz Sequences

Fri 9:45

Lori Alvin

Furman University

We investigate unimodal maps whose kneading sequences have a nice structure. We say that a kneading sequence $\mathcal{K}(f)$ has *substitutive structure* if there exists a substitution $\theta : \mathcal{A} \to \mathcal{A}^+$ with fixed point $\mathbf{w} = \lim_{n\to\infty} \theta^n(a)$ (for some $a \in \mathcal{A}$) and a rule $\varphi : \mathcal{A} \to \{0,1\}^+$ such that $\mathcal{K}(f) = \varphi(\mathbf{w})$. We study the relationships between various dynamical properties of the unimodal map and dynamical properties of the substitution shift. In particular, we discuss conditions where the sequence $\mathcal{K}(f)$ is Toeplitz (i.e., the turning point c is regularly recurrent).

Statistical and Non-Statistical Dynamics

Fri 10:30

Stefano Luzzatto

ICTP Trieste

We say that a dynamical systems is 'statistical' if the asymptotic distribution of almost all orbits in space converges to some probability distribution. This is what we intuitively expects such as when we flip a coin and the statistics converge to a 50-50 distribution. However there are some interesting examples in which the statistics of typical orbits does not converge, in the sense that the statistics depends on the time scale. I will give some precise definitions and examples.

Long-range order, mean equicontinuity and amorphic complexity

Fri 11:15

Maik Gröger

Jagiellonian University

Long-range order, mean equicontinuity and amorphic complexity Studying notions of long-rage order in symbolic and tiling dynamical systems via dynamical invariants has a long history which goes back to the seminal work of Hedlund and Morse. In this talk, I want to give an overview about the low-complexity notions of mean equicontinuity and amorphic complexity and how they intertwine. I will put a particular focus on classes of examples where amorphic complexity was explicitly calculated and the methods which were used for doing this. In part, this is joint work with G. Fuhrmann, T. Jäger, E. Krawczyk and D. Kwietniak.

Fri 13:30

Quotient of dynamical systems

Goran Erceg
University of Split

Let (X, f) be a dynamical system. Using an equivalence relation $\sim$ on X, we introduce the quotient $(X/_\sim, f^\star)$ of the dynamical system (X, f). We give new results about sensitive dependence on initial conditions of $(X/_\sim, f^\star)$, transitivity of $(X/_\sim, f^\star)$, and periodic points in $(X/_\sim, f^\star)$. We use these results to study chaotic functions on the Cantor fan and the Lelek fan.

This is joint work with Iztok Banič, Judy Kennedy, Chris Mouron, and Van Nall.

Fri 14:15

A note on finite coarse shape groups

Ivan Jelić
University of Split

We investigate properties concerning some recently introduced finite coarse shape invariants – the k-th finite coarse shape group of a pointed topological space and the k-th relative finite coarse shape group of a pointed topological pair. We define the notion of finite coarse shape group sequence of a pointed topological pair (X, X_0, x_0) as an analogue of homotopy and (coarse) shape group sequences and show that, for any pointed topological pair, the corresponding finite coarse shape group sequence is a chain. On the other hand, we construct an example of a pointed pair of metric continua whose finite coarse shape group sequence fails to be exact. Finally, using the aforementioned pair of metric continua together with a pointed dyadic solenoid, we show that finite coarse shape groups, in general, differ from both shape and coarse shape groups.

This is joint work with Nikola Koceić-Bilan.

Fri 15:00

End-point-generated smooth fans

Rene Gril Rogina
University of Maribor

We define end-point-generated smooth fans and give known examples. We also define smooth combs and use them to answer previously open problems about end-point-generated smooth fans as well as construct an uncountable family of such fans. This is joint work with Will Brian of UNC Charlotte.

On limit sets in hyperspace of continua in dimension one

Fri 15:30

Domagoj Jelić
University of Split

Whenever we are given a selfmap f of a compact metric space X, we can associate with it the induced mappings $\overline{f}$ and $\tilde{f}$ on the hyperspace 2^X of compact subsets of X and the hyperspace $C(X)$ of continua in X, respectively, both defined in a natural way.

In this talk we discuss and provide full description of the structure of ω-limit sets of induced system $\left(C(G), \tilde{f}\right)$, where f is a selfmap of a topological graph G.

This result extends previous results obtained first for much simpler cases of compact interval and topological tree by completely different tools.

The talk is based on a joint work with Piotr Oprocha.

Author Index

Chris Mouron, 4

Domagoj Jelić, 7

Goran Erceg, 6

Ivan Jelić, 6

Jakub Tomaszewski, 4
Jan P. Boroński, 1
Judy Kennedy, 2

Lori Alvin, 5

Maik Gröger, 5

Michał Kowalewski, 4

Nikola Koceić-Bilan, 3

Piotr Oprocha, 1

Rene Gril Rogina, 6

Sonja Štimac, 2
Stefano Luzzatto, 5

Udayan B. Darji, 2

Van Nall, 2