The 2nd Maribor Workshop on

Topological Dynamics, Continuum Theory, and related topics

Faculty of Natural Sciences and Mathematics
University of Maribor
Slovenia

3 - 7 February 2025

We cordially welcome you to the 2nd Maribor Workshop on Topological Dynamics, Continuum Theory and related topics! This edition of the workshop is special because we are commemorating the 50th anniversary of the founding of University of Maribor. We wish you an enjoyable and fruitful week ahead! And last but not least: we appeal to all the participants to help us make, with your enthusiasm and willingness to attend, this workshop a continuous annual event also in the years ahead. In particular, if you enjoy the edition of the workshop also this year, please do ask around when next year's event is going to happen - this will motivate us to actually organize it again!

Figure 1: University of Maribor

All talks and collaboration activities will take place in the lecture room 0/103 at the Faculty of Natural Sciences and Mathematics, Koroška cesta 160, 2000 Maribor. The conference dinner will take place on Thursday Feb 6 at 19.00 at the restaurant La Cantina in Maribor city center. On Wednesday Feb 5 we will have a guided tour of Maribor.

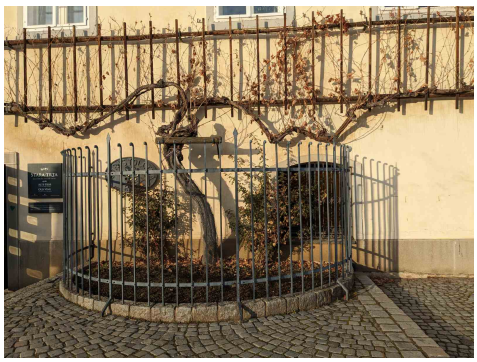

Figure 2: Old Vine

We wish you an enjoyable and fruitful workshop!

Organizers:
Lori Alvin
Iztok Banič
Jernej Činč
Matevž Črepnjak

Rene Gril Rogina
Teja Kac
Tina Sovič

The conference is supported by Slovenian Research and Innovation Agency (ARIS) project J1-4632.

Schedule

All talks are in the room 0/103.

Tuesday Feb 4 2025

9:00 – 9:55	Dominik Kweitniak
9:55 – 10:35	Ivan Jelić
10:35 – 11:00	Coffee
11:00 – 11:55	Will Brian
11:55 – 12:35	Bryant Rosado Silva
12:35 – 14:00	Lunch
14:00 – 14:55	Udayan B. Darji
14:55 – 15:35	James Kelly
15:35 – 16:00	Coffee
16:00 – 16:55	Piotr Oprocha
16:55 – 17:35	Domagoj Jelić

Wednesday Feb 5 2025

9:00 – 9:55	Luka Boc Thaler
9:55 – 10:35	Michaela Záškolná
10:35 – 11:00	Coffee
11:00 – 11:55	Judy Kennedy
11:55 – 12:35	Goran Erceg
12:35 – 14:00	Lunch
14:00 – 14:55	Toby Hall
14:55 – 15:35	Jakub Tomaszewski
15:35 – 16:10	Michał Kowalewski

Thursday Feb 6 2025

9:00 – 9:55	Veronica Martinez de la Vega
9:55 – 10:35	Van Nall
10:35 – 11:00	Coffee
11:00 – 11:55	Chris Mouron
11:55 – 12:35	Ana Anušić
12:35 – 14:00	Lunch
14:00 – 14:55	Stefano Luzzatto
14:55 – 15:35	Veronika Rýžová
15:35 – 16:00	Coffee
16:00 – 16:55	Sonja Štimac
16:55 – 17:35	Kristian Killasa Kvaternik

Friday Feb 7 2025

9:00 – 9:55	Henk Bruin
9:55 – 10:35	Silvia Radinger
10:35 – 11:00	Coffee
11:00 – 11:55	Vlasta Matijević
11:55 – 12:35	Rubio Gunawan
12:35 – 14:00	Lunch
14:00 – 14:40	Siniša Miličić
14:40 – 15:20	Matea Jelić

Contents

Abstracts

Even the vague specification property implies density of ergodic measures

Tue 9:00

Dominik Kwietniak

Jagiellonian University in Krakow

We prove that if a topological dynamical system (X, T) is surjective and has the vague specification property, then its ergodic measures are dense in the space of all invariant measures. The vague specification property generalises Bowen's classical specification property and encompasses majority of extensions of the specification property introduced so far.

The proof proceeds by first considering the natural extension X_T of (X, T) as a subsystem of the shift action on the space $X^{\mathbb{Z}}$ of X-valued biinfinite sequences. We then construct a sequence of subsystems of $X^{\mathbb{Z}}$ that approximate X_T in the Hausdorff metric induced by the metric compatible with the product topology on $X^{\mathbb{Z}}$. Approximating subsystems consist of δ-chains for δ decreasing to 0. We show that chain mixing implies that each approximating system possesses the classical periodic specification property. Furthermore, we use the vague specification to prove that our approximating subsystems of $X^{\mathbb{Z}}$ converge to X_T in the Hausdorff metric induced the Besicovitch pseudometric. It follows that the simplices of invariant measures of these subsystems of δ-chains converge to the simplex of invariant measures of X_T with respect to a generalised version of Ornstein's $\bar{d}$ metric. What is more, the density of ergodic measures is preserved in the limit. The proof concludes by observing that the simplices of invariant measures for X_T and (X, T) coincide.

The approximation technique developed in this paper appears to be of independent interest.

This is joint work with Damla Buldağ and Bhishan Jacelon.

Tue 9:55

Specification properties of CR-dynamical systems

Ivan Jelić

University of Split

We will recall the definition and basic properties of the notion of the specification property in the case of a standard topological dynamical system (X, f). We will then define a CR-dynamical system (X, F) (here X is a compact metric space and F is a closed relation on X) and introduce various generalizations of the specification property for this type of dynamical system. More precisely, we will introduce and investigate the notions of (strong/weak) specification property and compare them with their "initial" versions.

Tue 11:00

Endpoint-homogeneous fans

Will Brian

UNC Charlotte

A fan F is endpoint-homogeneous if for any two endpoints e and e' of F, there is a homeomorphism $h : F \to F$ such that $h(e) = e'$. I will share some recent results and questions regarding endpoint-homogeneous fans. This is joint work with Rene Gril Rogina.

Tue 11:55

Topological characterization of the generic maximal chain of the generalized Ważewski dendrites

Bryant Rosado Silva

Charles University

Let W be a generalized Ważewski dendrite. In this talk, we describe some properties of chains in the hyperspace $\mathrm{MOA}(W)$ of maximal chains of the hyperspace of subcontinua of W that yield comeager sets of chains. One of these properties is that every non-degenerate element of the chain is homeomorphic to W. Then, we give a topological characterization of a chain whose orbit under the action of the homeomorphism group on $\mathrm{MOA}(W)$ is a comeager set. This is a joint work with Benjamin Vejnar.

Dust to Fences

Tue 14:00

Udayan B. Darji

University of Louisville

In this talk we develop a very general method for raising maps of the Cantor space to various fences such as the Hairy Arc and Fraïssé Fence, while preserving the dynamics of the base homeomorphism of the Cantor space. As simple corollaries we obtain that Lelek fan admits maps with various dynamical properties. This is joint work with Jernej Činč and Benjamin Vejnar.

Dynamics of shifts on a compact, countable alphabet

Tue 14:55

James Kelly

Christopher Newport University

We investigate the dynamics of set-valued functions on compact, countable domains by way of the shifts spaces they induce. In this context we characterize topological transitivity and topological mixing, and we give results on topological entropy and shadowing.

On invariant sets with vanishing derivative and Cantor set dynamics

Tue 16:00

Piotr Oprocha

AGH University of Krakow

The aim of this talk is to characterize all dynamical systems on Cantor set that can be embedded in interval with vanishing derivative over it.

Starting motivation for this study is an old question whether invariant subset $C \subseteq [0,1]$ on which derivative of interval map f vanishes must contain a periodic point, which was recently answered in the negative by Ciesielski and Jasinski.

This talk is based on joint work with Silvere Gangloff.

Tue 16:55

A note on α-limit sets in the hyperspace of continua in dimension one

Domagoj Jelić

University of Split

In this talk, we will give a brief overview of two intriguing concepts in dynamics: namely hyperspace maps and α-limit sets.

Given a selfmap f of a compact metric space X, we can associate with it the induced mappings $\overline{f}$ and $\tilde{f}$ on the hyperspace 2^X of compact subsets of X and the hyperspace $C(X)$ of continua in X, respectively, both defined in a natural way. In this framework, it is interesting to understand the relation between the individual and collective dynamics.

Another concept of our interest will be that of the backward dynamics of the given system, which we study through the so-called α-limit sets. One way to define this notion, dual to that of $\omega-$limit set, is via backward branches of points in the system. A backward branch of a point $x \in X$ is any sequence $(x_i)_{i\leq 0}$, where $x_0 = x$ and $f(x_i) = x_{i+1}$ for each $i < 0$. By α-limit set of a backward branch $(x_i)_{i\leq 0}$ we mean the set $\alpha\left((x_i)_{i\leq 0}\right)$ consisting of all the accumulation points of $(x_i)_{i\leq 0}$.

We conclude by discussing which sets can be obtained as α-limit sets of backward branches in the induced system $\left(C(I), \tilde{f}\right)$, where f is a selfmap of a compact interval $I = [0, 1]$.

The talk is based on a joint work in progress with Piotr Oprocha.

Wed 9:00

Dynamics of skew-products tangent to the identity

Luka Boc Thaler

University of Ljubljana

We will look at the family maps $P_b(z, w) = (z + z^2, w + w^2 + bz^2)$ with $b \in \mathbb{C}$. Already such simple family of maps exhibits a rich variety of dynamical behaviors, in particular there are values $b > \frac{1}{4}$ for which a map has a wandering domain. We will see that there is a sequence of parameters b_n converging to $\frac{1}{4}$, for which maps P_{b_n} are not topologically conjugate to each other, which in particular this implies that the family P_b is not structurally stable at $b = \frac{1}{4}$.

Transitive compact systems on the interval

Wed 9:55

Michaela Záškolná
Silesian University of Opava

In the year 2016 Wen Huang, Danylo Khilko, Sergii Kolyada and Guohua Zhang published an article on dynamical compactness and sensitivity where they introduced the concept of $\omega_{\mathcal{N}_T}$-limit sets and transitive compactness to connect the Auslander point dynamics with topological transitivity. In this talk we study the properties of $\omega_{\mathcal{N}_T}$-limit sets of chaotic dynamical systems (X, T) given by a compact metric space X and a surjective map $T : X \to X$ and show that if we restrict ourselves to interval mappings, then transitive compactness and weak mixing are equivalent. Lastly, we discuss open problems.

Non-smooth fans

Wed 11:00

Judy Kennedy
Lamar University

We discuss our recent work on non-smooth fans. This is joint work with Iztok Banič, Goran Erceg, and Ivan Jelić.

Uncountable family of Lelek-like fans

Wed 11:55

Goran Erceg
University of Split

Defining an appropriate equivalence relation on a Lelek fan L we construct an uncountable family of pairwise non-homeomorphic Lelek-like fans. In this talk plan is to explain the construction of that family.

Folding maps

Wed 14:00

Toby Hall
University of Liverpool

This is joint work in progress with André de Carvalho and Daniel Meyer.

Take a square S of paper. Fold the right half over the left half, and then the back half over the front half. This defines a map $f : S \to S$, which is a quotient of a Lattès map of the Riemann sphere.

Fattening S into a cube $C = S \times [0, 1]$ which is squeezed by a factor 4 in the vertical direction, we can turn f into a "3-dimensional pseudo-Anosov homeomorphism" F of the 3-sphere $C/_R$ where R is an equivalence relation on C.

This is a simple case of a more general construction. I'll describe this case in some detail, and talk briefly about the general case.

Wed 14:55

On the MME for dendrite maps

Jakub Tomaszewski
AGH University of Krakow

In the talk we will focus on countably piecewise linear maps on dendrite spaces. We will develop and work on a notion of "markovian" dynamical system, which will be a natural extension of markov maps on topological graphs onto dendrites. Using the framework build on results by Vere-Jones, Gurevic and Ruette, we will show the necessary conditions for markovian maps to have measure of maximal entropy, discussing also the characteristics of such measure.

Wed 15:35

Connections between generalized sin(1/x)-type continua and ClassW

Michał Kowalewski
AGH University of Krakow

We call X a tranched graph if we can map it by some monotone map ϕ onto a topological graph 1-1 on a dense subset of X. Futhermore, if any subcontinuum of a fiber can be approximated by a preimage of an arc by ϕ, we call X a generalized sin(1/x)-type continuum.

In study of generalized sin(1/x)-type continua one finds, that topology of non-degenerate fibers (non-degenerate subcontinua of X that collapse to a point in the topological graph) is crucial. A very useful tool in problem of characterizing generalized sin(1/x)-type continua is well researched ClassW.

I will present the main properties of sin(1/x)-type continua, define ClassW, and describe how the two intertwine.

Thu 9:00

Dynamics on Continua and Hyperspaces

Verónica Martínez de la Vega y Mansilla
Universidad Nacional Autónoma de México

I will talk about some results on dynamics of maps on certain classes of continua and dynamics of induced maps of hyperspaces. The results I am going to talk about is still an ongoing project, which I hope has certain ramifications we can discuss.

Turbulence and Entropy of shift maps on Mahavier products

Thu 9:55

Van Nall
University of Richmond

A continuous function $f : X \to X$ on a compact space X is turbulent if there are disjoint nonempty closed sets A and B in X such that $A \cup B \subset f(A) \cap f(B)$. Misiurewicz proved that if $X = [0, 1]$ then f has positive entropy if and only if for some n, f^n is turbulent. The same is not true for all compact topological spaces X or even all one dimensional continua. For example there is a homeomorphism on the Cantor fan that has positive entropy. We will consider positive entropy shift maps on Mahavier products constructed with a closed relation on $[0, 1]$.

Dynamics on Hereditarily Decomposable Tree-like Continuum

Thu 11:00

Christopher Mouron
Rhodes College

In this talk give an example of a hereditarily decomposable tree-like continuum that admits homeomorphisms that have the following dynamic properties: mixing, the specification property, and continuum-wise turbulence. I will also give results about topological properties (or lack of properties) that prevent hereditarily decomposable tree-like continuum from admitting homeomorphisms with some of the previous properties.

Accessible sets in planar embeddings of tree-like continua

Thu 11:55

Ana Anušić
University of Zagreb

I will give an overview of recent results (joint with A. Ammerlaan and L. Hoehn) on planar embeddings of arc-like continua and generalizations, focusing on the recent solution of the Nadler-Quinn problem (1972). Given an arc-like continuum X, and a point x in X, we proved that there is a planar embedding of X in which x is an accessible point. The main idea in the proof is constructing different representations of X as an inverse limit on intervals. Such techniques were further generalized to give conditions when non-degenerate sets of X can be simultaneously embedded accessible. As an example, for every countable subset S of the Knaster continuum, we obtain a planar embedding in which S is accessible, answering a question of Debski and Tymchatin.

We also show that for every positive integer n, the Knaster continuum can be realized as an attractor of a planar homeomorphism such that (at least) n of its composants are accessible. I will shortly mention our current work in which we are generalizing the methods to tree-like continua. In particular, we constructed new planar embeddings of Ingram's atriodic tree-like continuum with positive span, answering a question of J. Mayer.

Thu 14:00 **Computer Assisted Computations in the Quadratic Family**

Stefano Luzzatto
ICTP Trieste

It has been known already for 25 years that almost every parameter in the one-dimensional quadratic family is either "regular" or "stochastic". Interesting the set of regular and stochastic parameters have very different topological structures: the set of regular parameters is open and dense in the parameter space, and therefore the set of stochastic parameters is nowhere dense. However both of them have positive Lebesgue measure.

Interestingly, very little is know about the actual value of the measure of the two sets, and for 90% of parameters it is not know if they are regular or stochastic. In this talk I will explain more in detail the problem and discuss some work in progress which uses rigorous numerics and computer-assisted computations to address this question.

Thu 14:55 **On one of Birkhoff's theorems for backward limit points**

Veronika Rýžová
Silesian University of Opava

In 1927 George Birkhoff in his book Dynamical Systems presented a theorem that describes the behaviour of trajectories outside of a set of non-wandering points on an arbitrary compacta. Much later in 1960s Sharkovsky followed up on Birkhoff's work and published even stronger result, this time focusing on the set of omega limit points for interval maps. We formulate similar statement for a neighbourhood of a set of different types of backward limit points for maps of the interval.

Thu 16:00 **Two properties of the Hénon maps with strange attractors**

Sonja Štimac
University of Zagreb

Recently, Jan Boroński and I proved that the H énon maps on their strange attractors (for the Wang-Young parameters WY) are topologically conjugate to the shift homeomorphisms on the inverse limits of certain maps on densely branched dendrites T and that this model is optimal. I will talk about a new development where we proved that every branch point of T has order three and represents a tangential intersection of the boundary of D and a stable manifold. As a corollary, we obtain that T is a unique topological object, the same for all parameters in WY, namely the universal dendrite of order three. This is joint work with Jan Boroński..

Zero entropy and basin of attraction for Lozi maps

Thu 16:55

Kristijan Kilassa Kvaternik
University of Zagreb

We consider the dynamics of the two-parameter Lozi family of planar homeomorphisms, together with the stable and unstable manifold of the hyperbolic fixed point X of that family in the first quadrant. We introduce a specific region in the parameter space for which the period-two orbit is attracting and there are no homoclinic points for X. In this region, we observe the set ℓ of accumulation points of the unstable manifold of X. We show that the Lozi map, restricted to the complement of ℓ, has zero topological entropy. In addition, we discuss the basin of attraction for the Lozi map determined by the stable manifold of the fixed point in the third quadrant.

On complex kneading theory

Fri 9:00

Henk Bruin
University of Vienna

Kneading theory is the symbolic dynamics of (unimodal) maps on the interval, but it can be extended to maps on other spaces, for example as quadratic maps on the complex plane. The chaotic dynamics takes place on the Julia set, but for the behaviour of the critical orbit, it suffices to look at a tree (or dendrite) called Hubbard tree. Problems solved include the admissibility question (i.e., which 0-1-sequences can be the kneading sequence of Hubbard trees), how to reconstruct the Hubbard tree from the kneading sequence, and where to locate kneading sequences on a Mandelbrot set.

This talk is based on joint papers with Dierk Schleicher and Alexandra Kaffl.

Rigidity of Toeplitz and Bratteli-Vershik Dynamical Systems

Fri 9:55

Silvia Radinger
University of Vienna

In this talk we will study measure-theoretical rigidity and partial rigidity for classes of Cantor dynamical systems including Toeplitz systems. The use of Bratteli-Vershik dynamical systems enables us to control the structure of invariant measures. Among other things, we will analyze different Toeplitz systems for their rigidity, show that there exist Toeplitz systems which have zero entropy and are not partially measure theoretically rigid with respect to any of its invariant measures and construct a Toeplitz system which has countably infinitely many ergodic invariant probability measures that are rigid with the same rigidity sequence. Further we show varying rigidity in a family of enumeration systems defined by a linear recursion.

This talk is based on joint work with Henk Bruin, Olena Karpel and Piotr Oprocha.

Fri 11:00

On non-commutative systems

Vlasta Matijević

University of Split

One fruitful and very old technique for studying topological spaces with "bad" local properties is to present them as limits of inverse systems of "nice" spaces, usually polyhedra. However, in the fifties of the past century it was noticed that this technique applied to compact Hausdorff non-metrizable spaces has some deficiencies. This led S. Mardešić and T. Watanabe in 1989 to introduce a more flexible kind of inverse systems of topological spaces, called approximate systems. The main idea was to abandon the rigid functorial requirement of commutativity of bonding maps and allow them to differ in a certain controlled way. Recently, in joint work with L. Rubin, in answering a question under which conditions Čech systems of topological spaces are approximate systems, a new class of non-commutative systems appeared. We called those systems delay-inverse systems since the commutativity law for bonding maps is fulfilled after some "delay". In this talk some old and some new results and examples of non-commutative systems will be presented.

Fri 11:55

Uniformly expanding full branch maps with attracting fixed point with full measure basin

Rubio Gunawan

ICTP Trieste

We prove the existence of a uniformly expanding full branch map $f : [0, 1] \to [0, 1]$, with C^∞ regularity on each branch, and a Milnor attracting fixed point. It has a countable partition $\mathcal{P}_1 = \{I_1, I_2, I_3, ...\}$, with the branches approaching 0 as $n \to \infty$. 0 is the attracting fixed point of our map. It is not an attractor in the classical sense because there is no neighborhood of 0 which is invariant and attracted to 0. But it is a Milnor attractor because for Lebesgue almost every point x, $\omega(x) = \{0\}$. This is also a nice example of a system with infinite entropy but very simple physical measure.

An intrinsic framing of the box dimension

Fri 14:00

Siniša Miličić
University of Pula

The concept of box dimension is typically introduced through the intuitive and geometric **box-counting framing**, which serves as a simple explanatory tool. However, the **Minkowski-Bouligand framing** is more efficient for computational purposes but relies on the presence of an ambient Euclidean space. In this work, we propose an alternative formulation of the box dimension that is grounded intrinsically in the appropriate-dimensional Hausdorff measure.

Given a Ahlfors regular space S of Hausdorff dimension d and the d-dimensional Hausdorff measure h^d, we define ϵ-dependent functions and measures (f_ϵ-boxdim-functions and f_ϵ-boxdim-measures) that encapsulate the scaling behavior of the space. Specifically, for an $\epsilon > 0$, an ϵ-boxdim-function $f_\epsilon : S \to \mathbb{R}$ satisfies:

$$\int_{B_\epsilon(x)} f_\epsilon dh^d = O(\epsilon^d), \text{ for all } x \in S.$$

From these functions, we derive the f_ϵ-boxdim-measure ν_{f_ϵ}, defined in terms of Radon-Nykodim derivative as

$$\frac{d\nu_{f_\epsilon}}{dh^d} = f_\epsilon.$$

We analyze the ϵ-asymptotics of such measures to explore its properties and establish connections with the Minkowski-Bouligand framework, the classical box dimension, and intrinsic characterizations of scaling behavior. Through this lens, we investigate classical examples of box dimensions.

Computable approximations of semicomputable chainable continua

Fri 14:40

Matea Jelić
University of Split

The central notion of this talk is the notion of a computable set in a topological space. While each computable set is semicomputable, the vice versa does not hold, i.e. there are semicomputable sets which are not computable. But, it turns out that some topological properties of a set S can ensure that the implication: S semicomputable $\Rightarrow$ S computable holds. The fact that it still does not hold in general raises the following question: under what conditions a semicomputable set S can be approximated by a computable subset with arbitrary precision? Any semicomputable continuum S in a computable topological space can be approximated by a computable subcontinuum by any given precision under condition that S is chainable and decomposable. In this talk we show that the assumption that S is decomposable can be replaced with the following assumption: S is chainable from a to b and a is a computable point. In that case we prove that S can be approximated by computable subcontinuum chainable from a to b', where b' is a computable point.

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